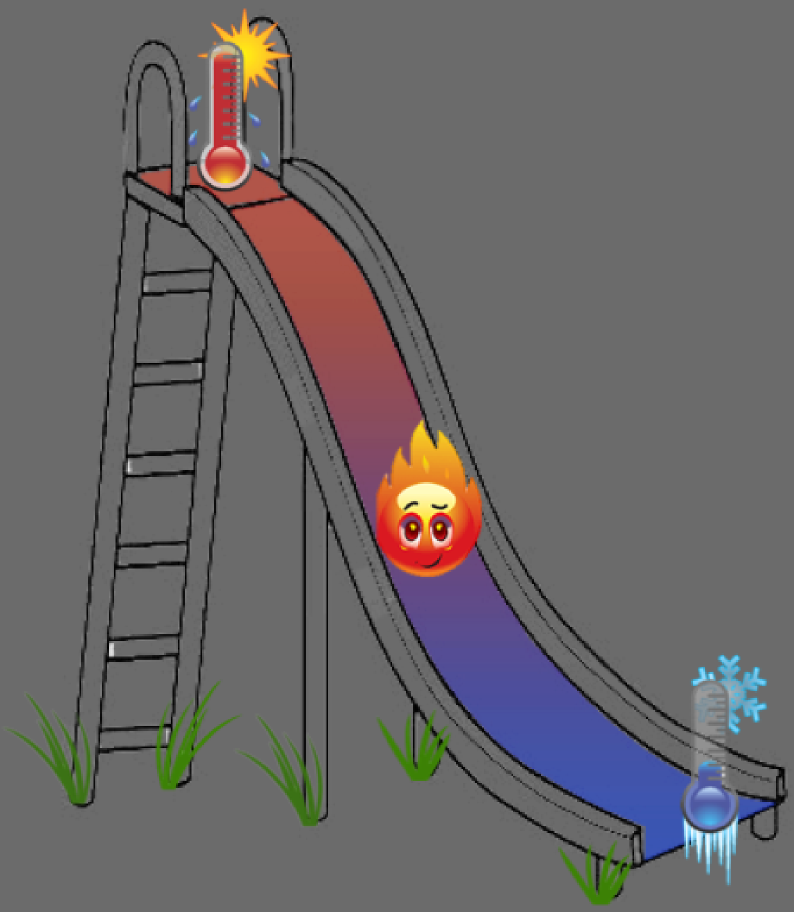
quantization of heat flow



Pendry's theory for 1D heat transport

1D channel
non-interacting particles

J B Pendry
Quantum limits to the flow of information and entropy
J. Phys. **A:** Math. Gen. **16** (1983) 2161-2171

thermal energy = temperature x entropy $\Delta Q = T \Delta S$

entropy = log 2 x information

universal upper limit of information transfer

hence, a limit of thermal conductance $KT < \kappa_0 T$

$$\frac{dJ_{th}}{dT} = K\,T$$

1D ballistic transport

quantum of thermal conductance

$$\frac{dJ_{th}}{dT} = \kappa_0 T$$

$$\kappa_0 = \frac{\pi^2 k_B^2}{3h}$$

$$\kappa_0 = 9.5 \times 10^{-13} W / K^2$$

$$J_{th} = \frac{1}{2}\kappa_0 (T^2 - T_0^2)$$

Wiedemann – Franz ballistic 1D channel

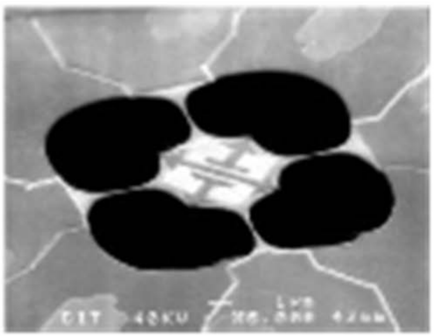
for non-interacting electrons

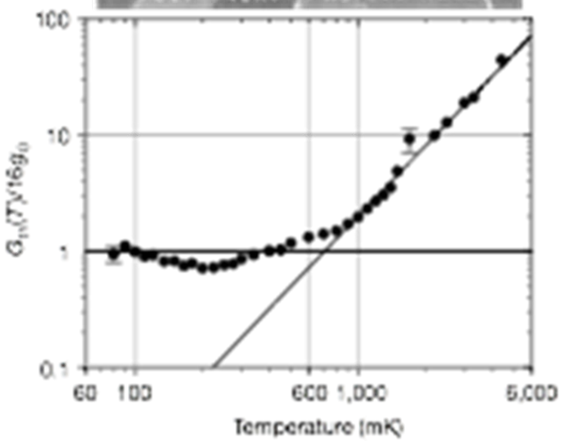
$$G_{th} = \kappa_0 T \qquad G_e = \frac{e^2}{h}$$

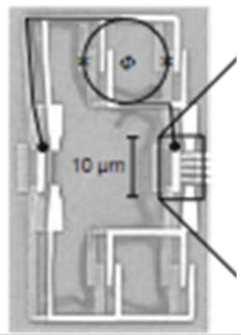
$$\frac{G_{th}}{G_e} = l_{Lorentz} T = \frac{\pi^2 k_B^2}{3e^2} T$$

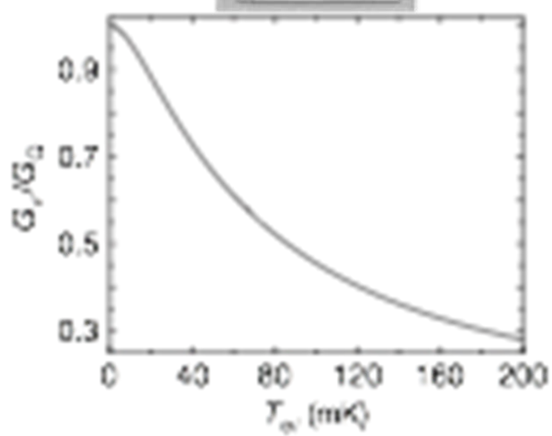
past experiments…. in accord with theory

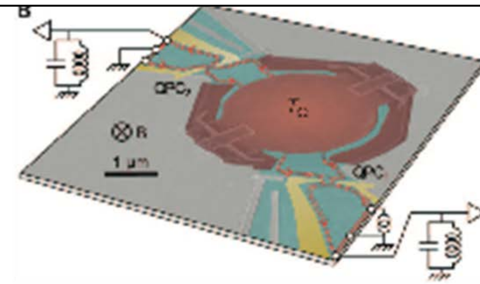
16 phonon modes Schwab et al, 2000	single photon mode Meschke et al, 2006	single electron mode Jezouin et al, 2013

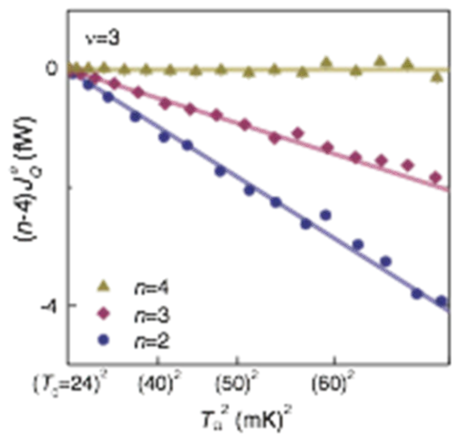












Wiidemann – Franz

Pendry's general theory of **non-interacting** particles in 1D

was extended for **interacting** particles

Kane, C. L. & Fisher, M. P. A.
Quantized thermal transport in the fractional quantum Hall effect
Phys. Rev. B **55,** 15832–15837 (1997)

interactions <u>should not</u> affect
the thermal conductance

$$K \leq \kappa_0$$

Wiedemann - Franz breaks down

our 1D interacting system……FQHE

without reproducible interference in fractional states…

thermal transport may distinguish

non abelian *vs* abelian

why thermal conductance *K* ?

- topological number, determined by *bulk wave - function*
- reveals the **NET** chirality of modes….down - up
- independent of *edge – reconstruction*….may add modes
- may provide proof of *non-abelian states* (w/ Majorana)

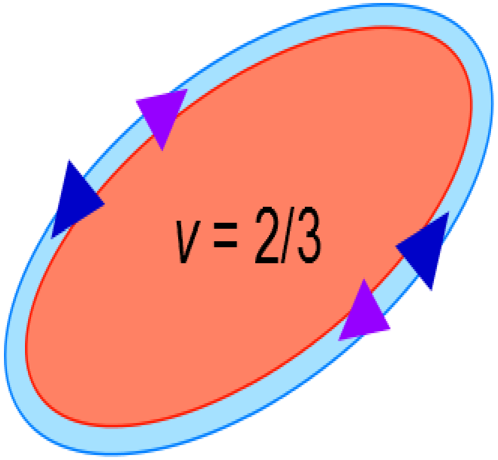
particle - like $\nu = 1/3$

$K = \kappa_0$

hole - like $\nu = 2/3$ *polarized*

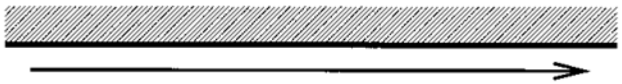
$\nu = 2/3 \;=\; 2/3 - \text{neutral}_{\text{upstream}}$

$K = 0$



Kane, C. L., Fisher, M. P. A. & Polchinski, J.
Randomness at the edge:
theory of quantum Hall transport at filling 2/3
Phys. Rev. Lett. **72**, 4129–4132 (1994)

κ in lowest *LL*... *Kane & Fisher* 1997

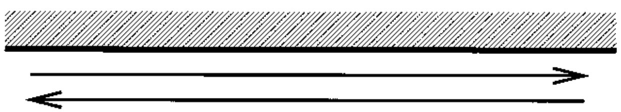


$\nu = 1/3 \rightarrow \kappa_0$

1 composite fermion mode

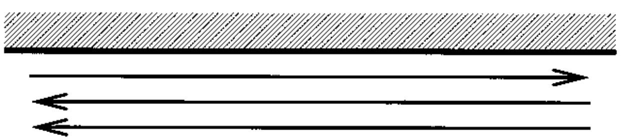
$\nu = 2/5 \rightarrow 2\kappa_0$

2 composite fermion modes



$\nu = 2/3 \rightarrow 0$

1 charge down + **1** neutral up

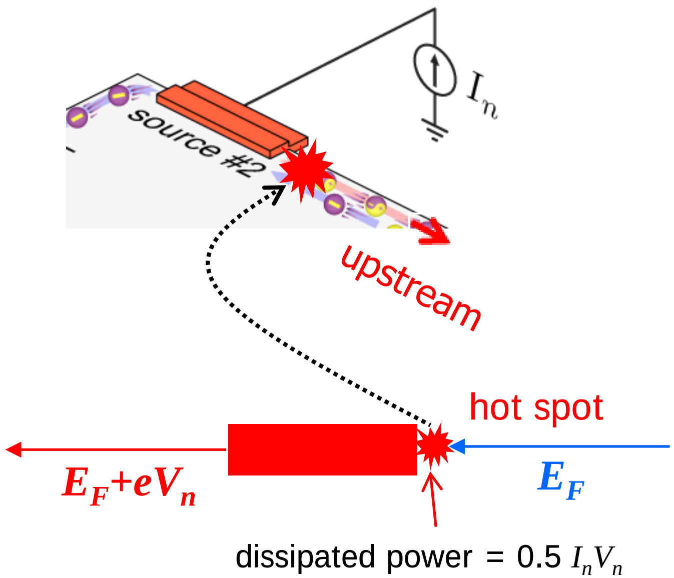


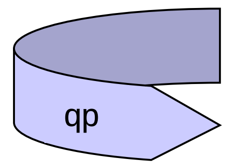
$\nu = 3/5 \rightarrow -\kappa_0$

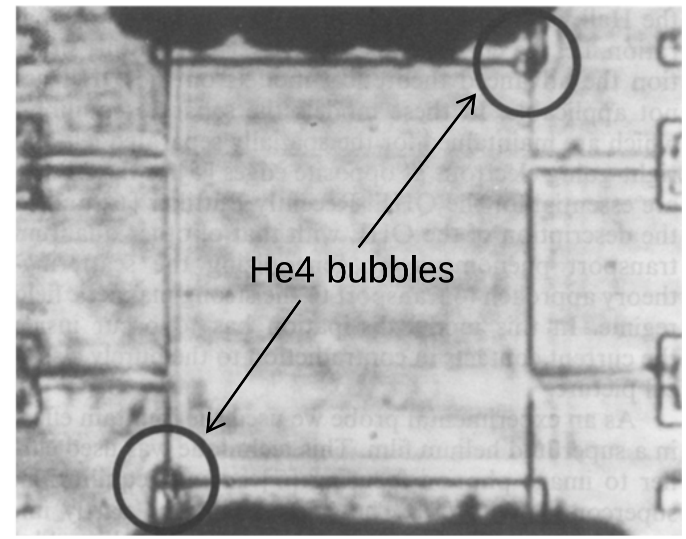
1 charge down + **2** neutral up

excitation of neutral modes *at ohmic contact*

'back side' of ohmic contact







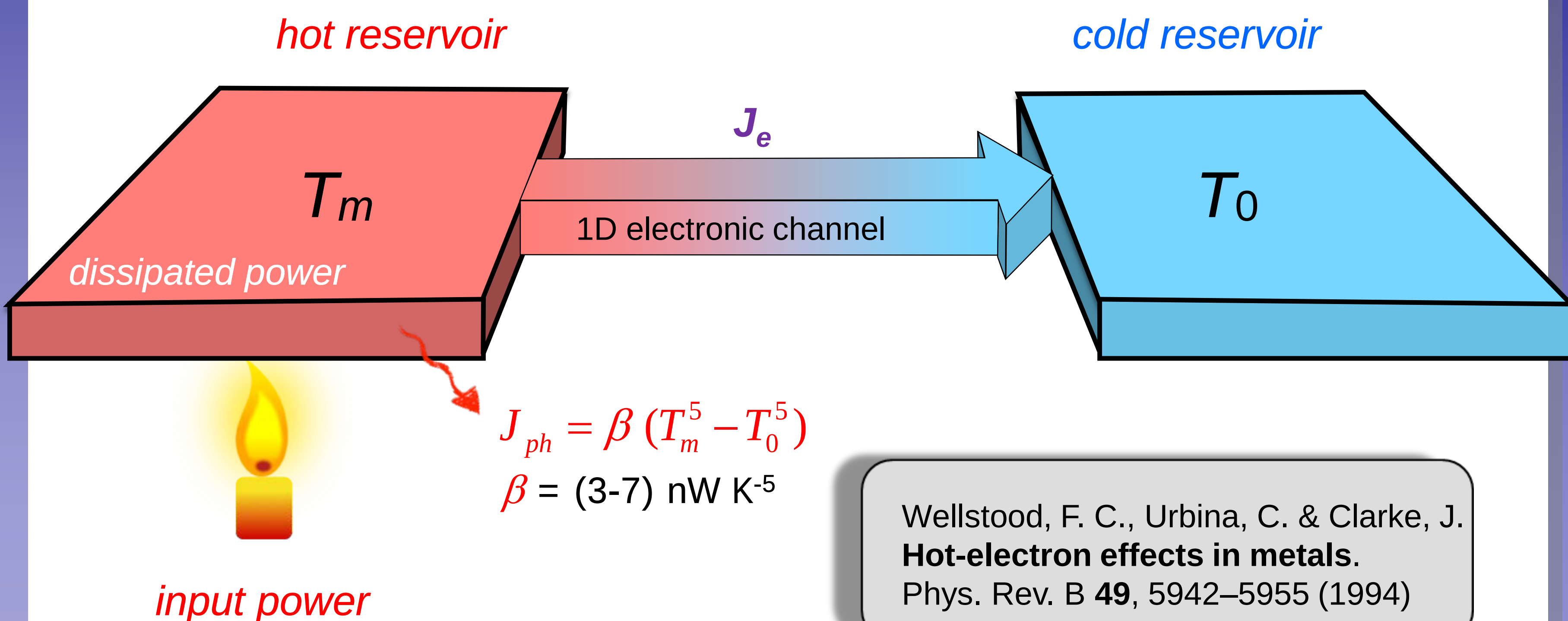
U. Klass, W. Dietsche, K. von Klitzing & K. Ploog
Image of the dissipation in gated quantum Hall effect samples
Surface Science **263**, 97-99 (1992)

the experiments

working principle

flow of dissipated power…..

$$J_{tot} = \underbrace{\frac{1}{2}\kappa_0 (T_m^2 - T_0^2)}_{\text{electrons}} + \underbrace{J_{ph}}_{\text{phonons}}$$

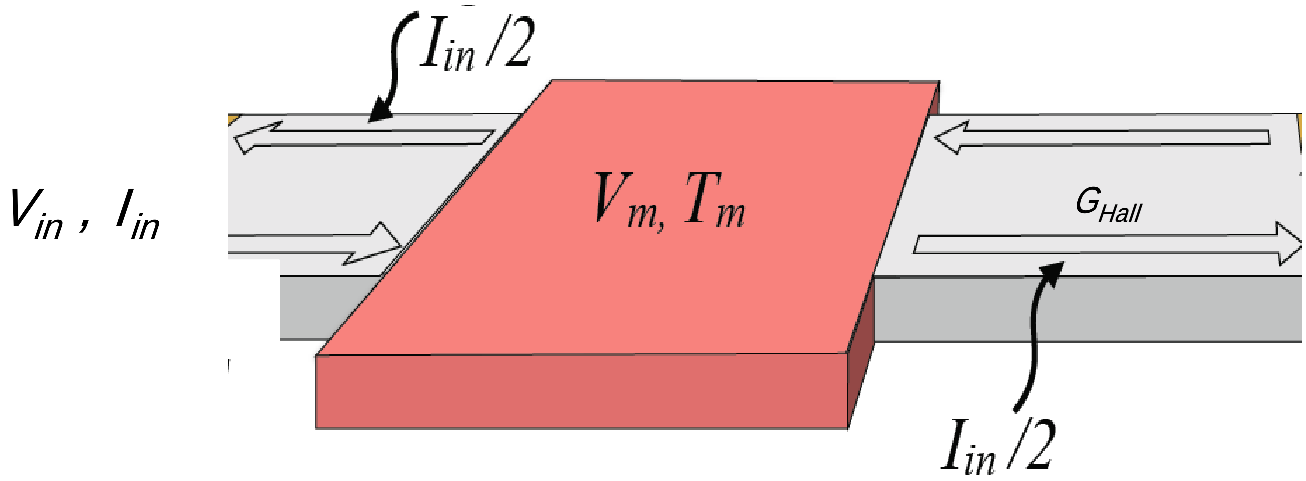


$$J_{ph} = \beta\,(T_m^5 - T_0^5)$$

β = (3-7) nW K^{-5}

Wellstood, F. C., Urbina, C. & Clarke, J. **Hot-electron effects in metals**. Phys. Rev. B **49**, 5942–5955 (1994)

replacing heating reservoir w/ current …



$$I_{out} = I_{in}/2 + I_{in}/2 = I_{in}$$

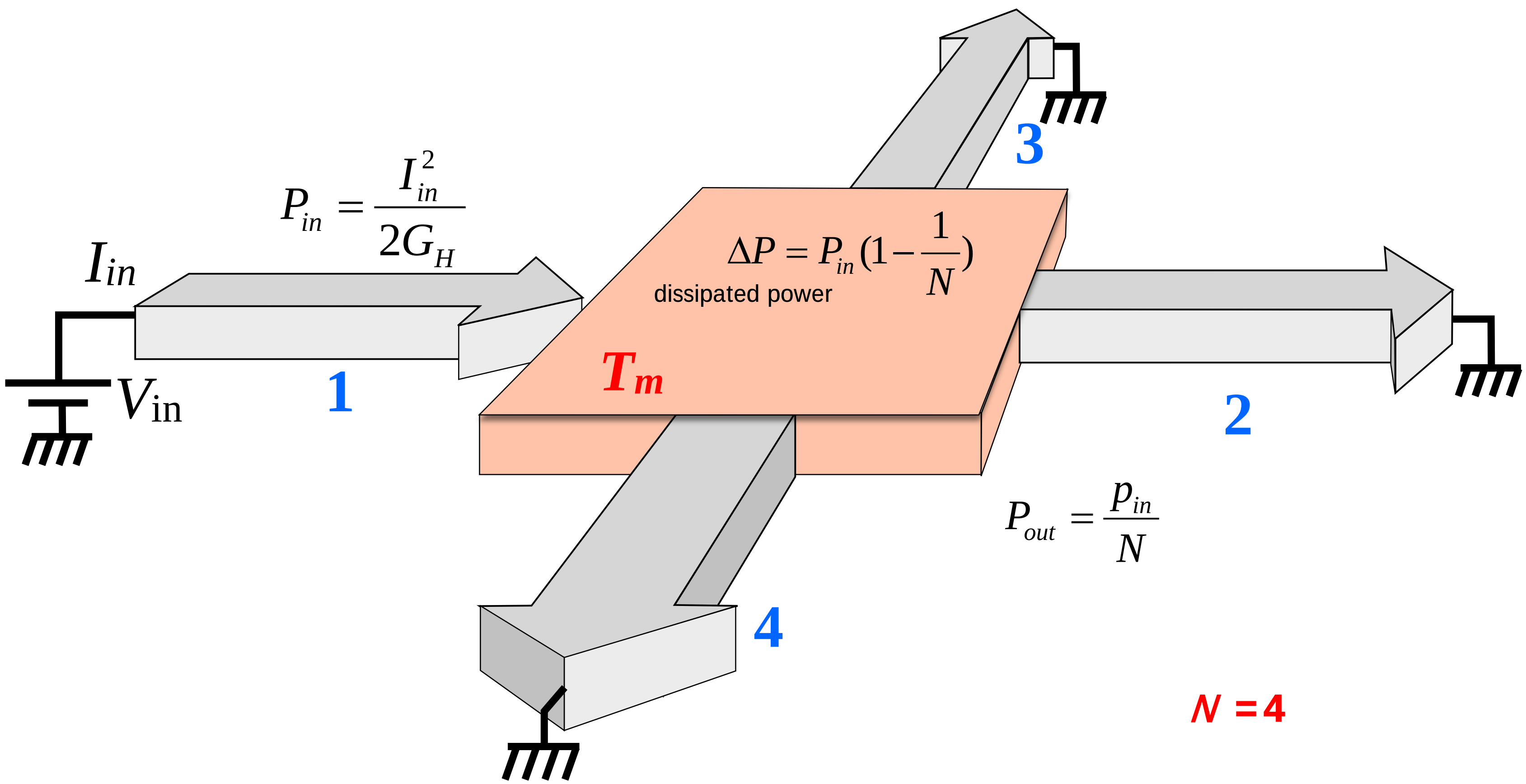
$$V_m = V_{in}/2$$

$$P_{out} = P_{in}/2$$

$$\Delta P = P_{in} - P_{out} = P_{in}/2$$

Jezouin *et al.* Science 2013

N – arm device



$N = 4$

we measure only...

$$\Delta P = J_{th}^{total} = 0.5\,K\,(T_m^2 - T_0^2) + \beta\,(T_m^5 - T_0^5)$$

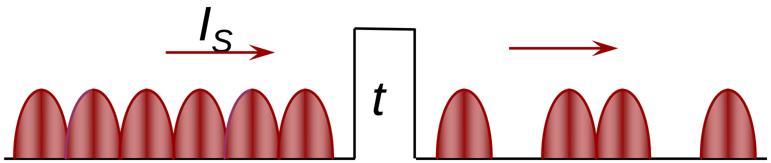
electron temperature in grounded contacts.....T_0

electron temperature in heated reservoir.......T_m

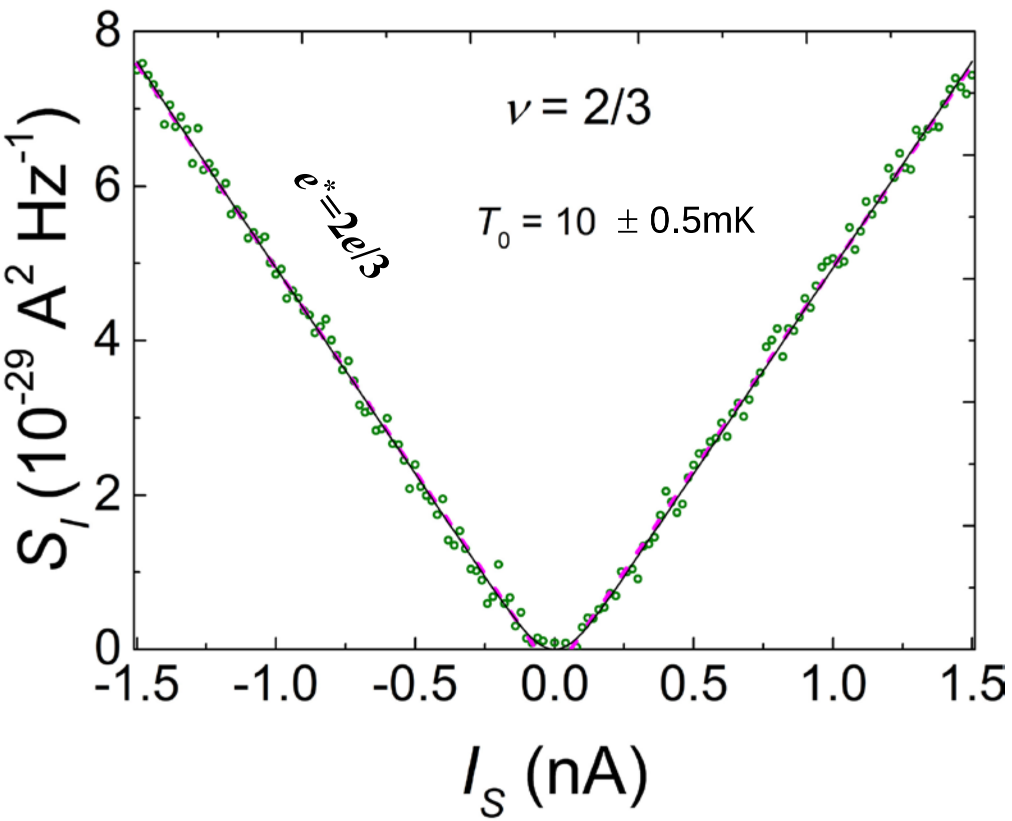
- small *T*.........phonon term irrelevant
- high *T*..........phonon term subtracted

K determined

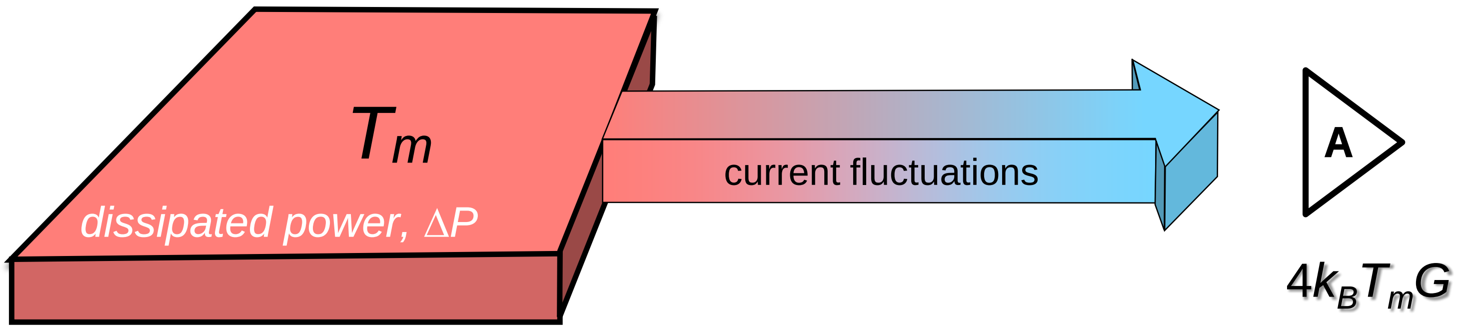
measuring T_0 ……*shot noise*



$$S_i(\omega \sim 0) = 2e^* I_S t(1-t) \cdot \Im(V_S, T) + 4k_B TG$$



measuring T_m ……*Johnson-Nyquist noise*

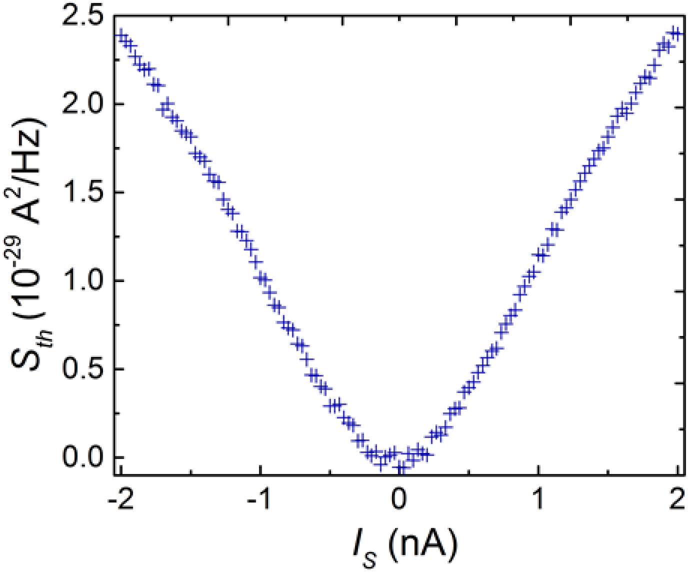


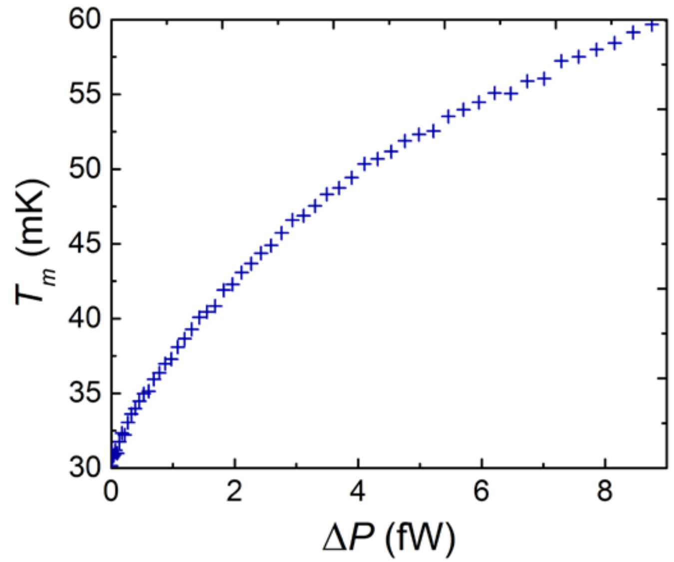
- modes leave contact with noise $4k_BT_mG$
- even if modes cool down with distance...

low frequency current fluctuations conserved

measuring T_m ……*excess Johnson-Nyquist noise*

measured noise





determining thermal conductance

calculated dissipated power $\Delta P = P_{in}(1 - \frac{1}{N_{arms}})$

phonons dissipation from 'floating reservoir' depends ONLY on its T_m

for T_m large….subtracting procedure

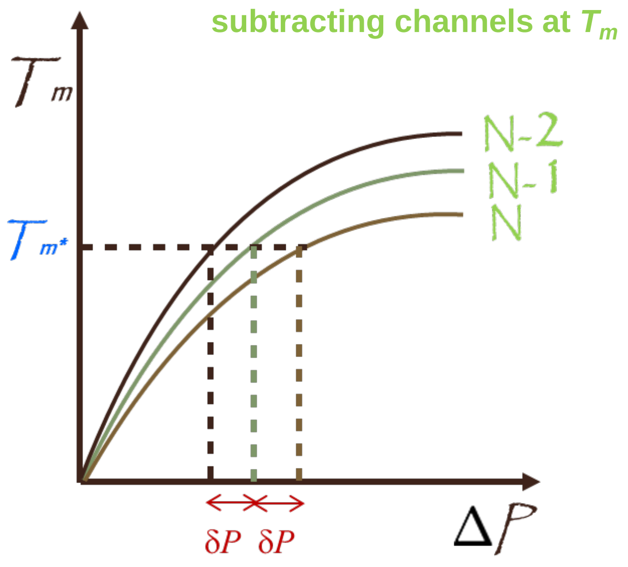
$$-\left[\begin{array}{l} \Delta P_{N_i} = \frac{1}{2} N_i * K(T_m^2 - T_0^2) + J_{ph}(T_m) \\ \Delta P_{N_j} = \frac{1}{2} N_j * K(T_m^2 - T_0^2) + J_{ph}(T_m) \end{array}\right.$$

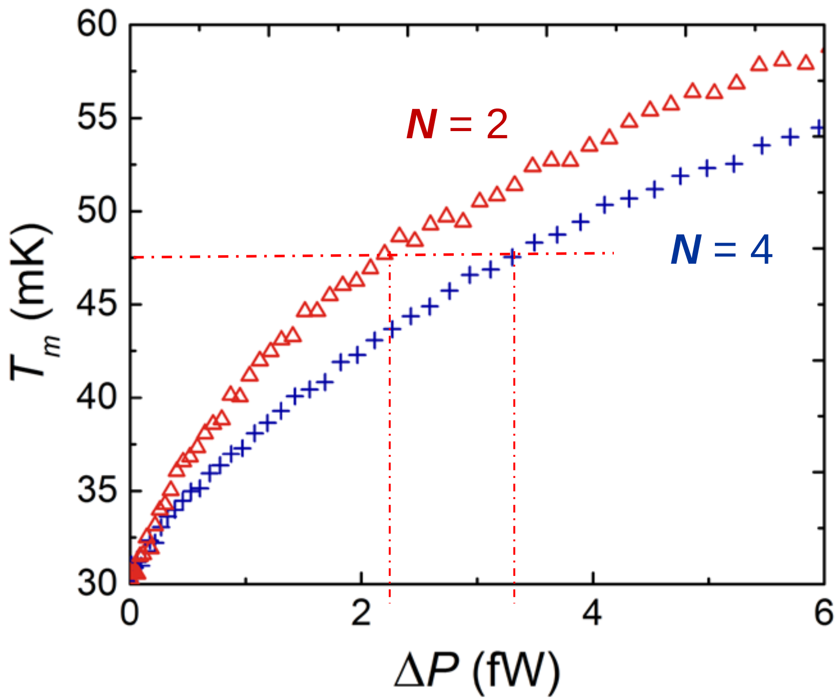
same T_m

$$\delta P_{N_i - N_j} = \frac{1}{2}(N_i - N_j) * K(T_m^2 - T_0^2)$$

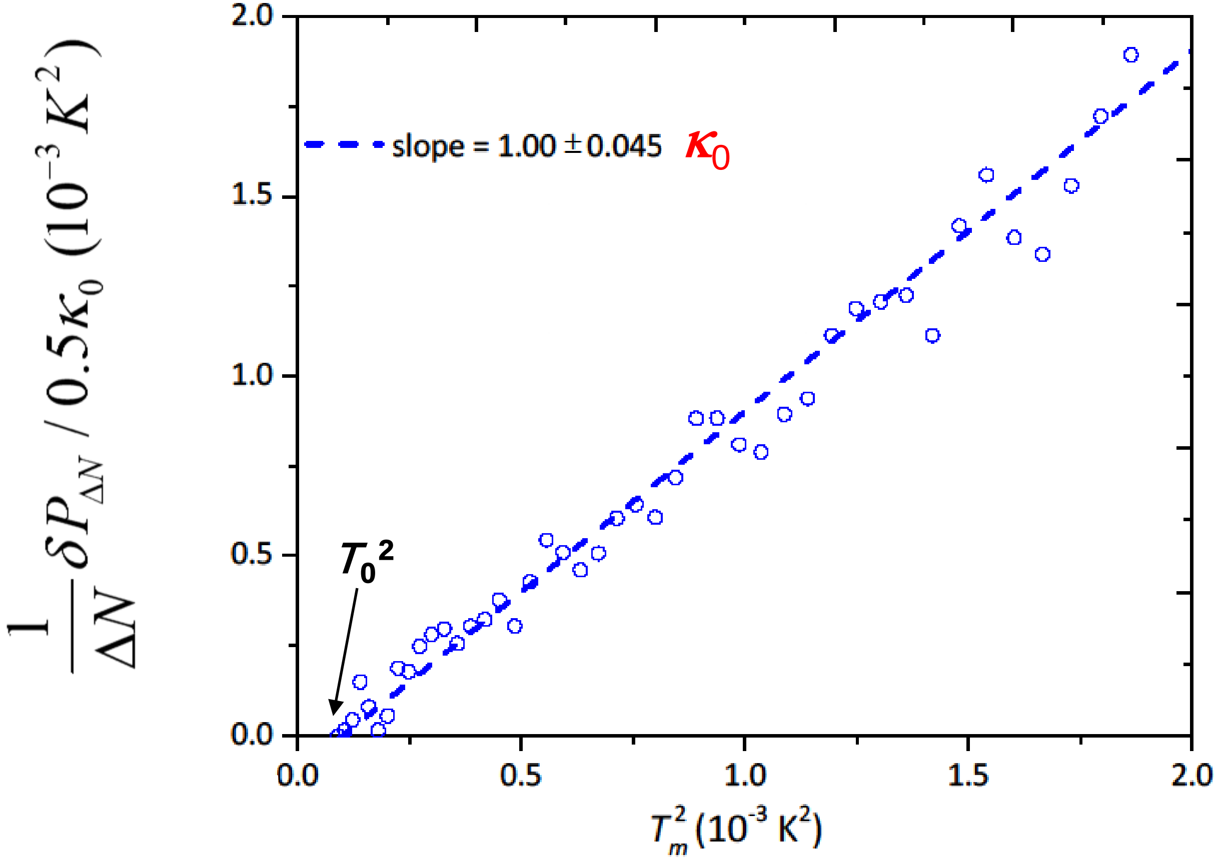
phonon contribution subtracted

subtracting phonon contribution

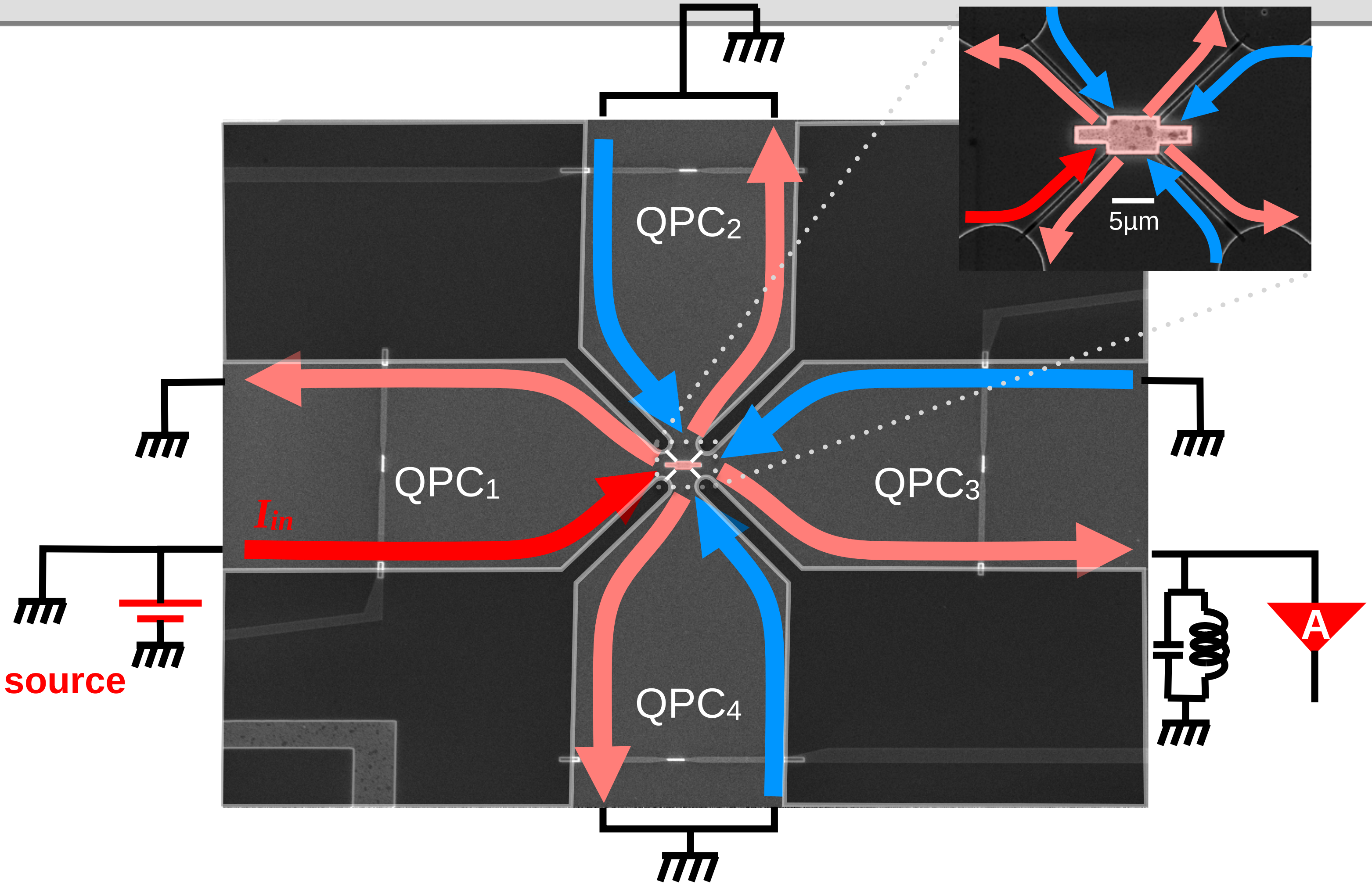




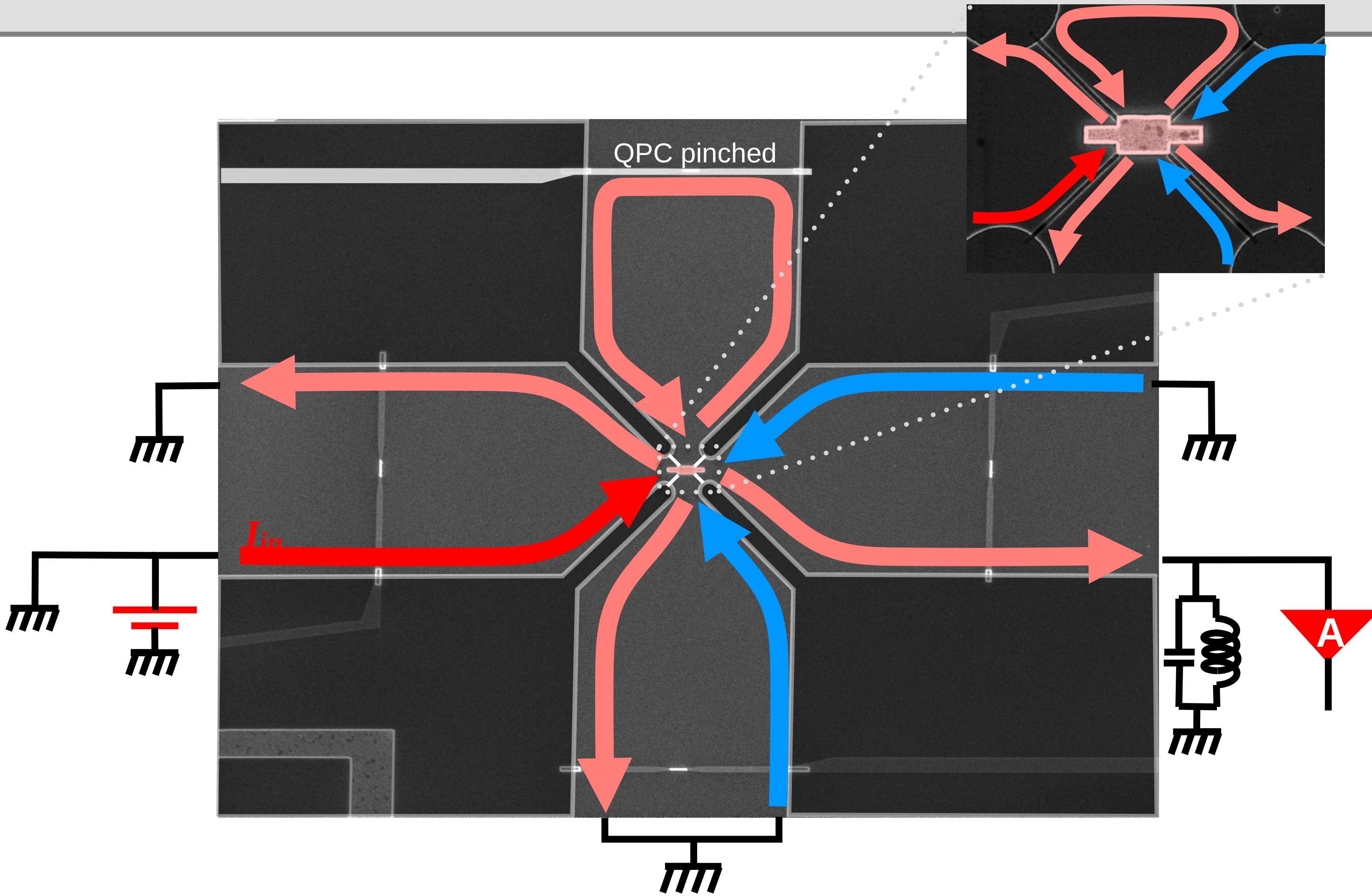
getting K / κ_0 ……an example



realization.......N = 4

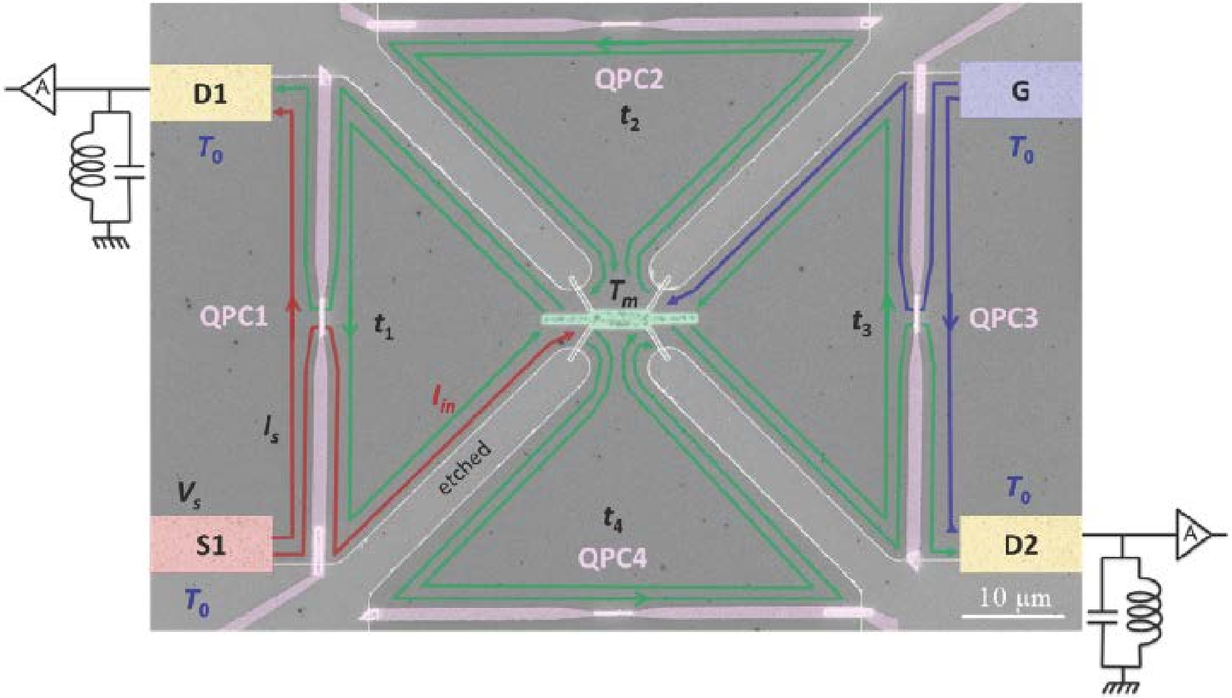


N = 3

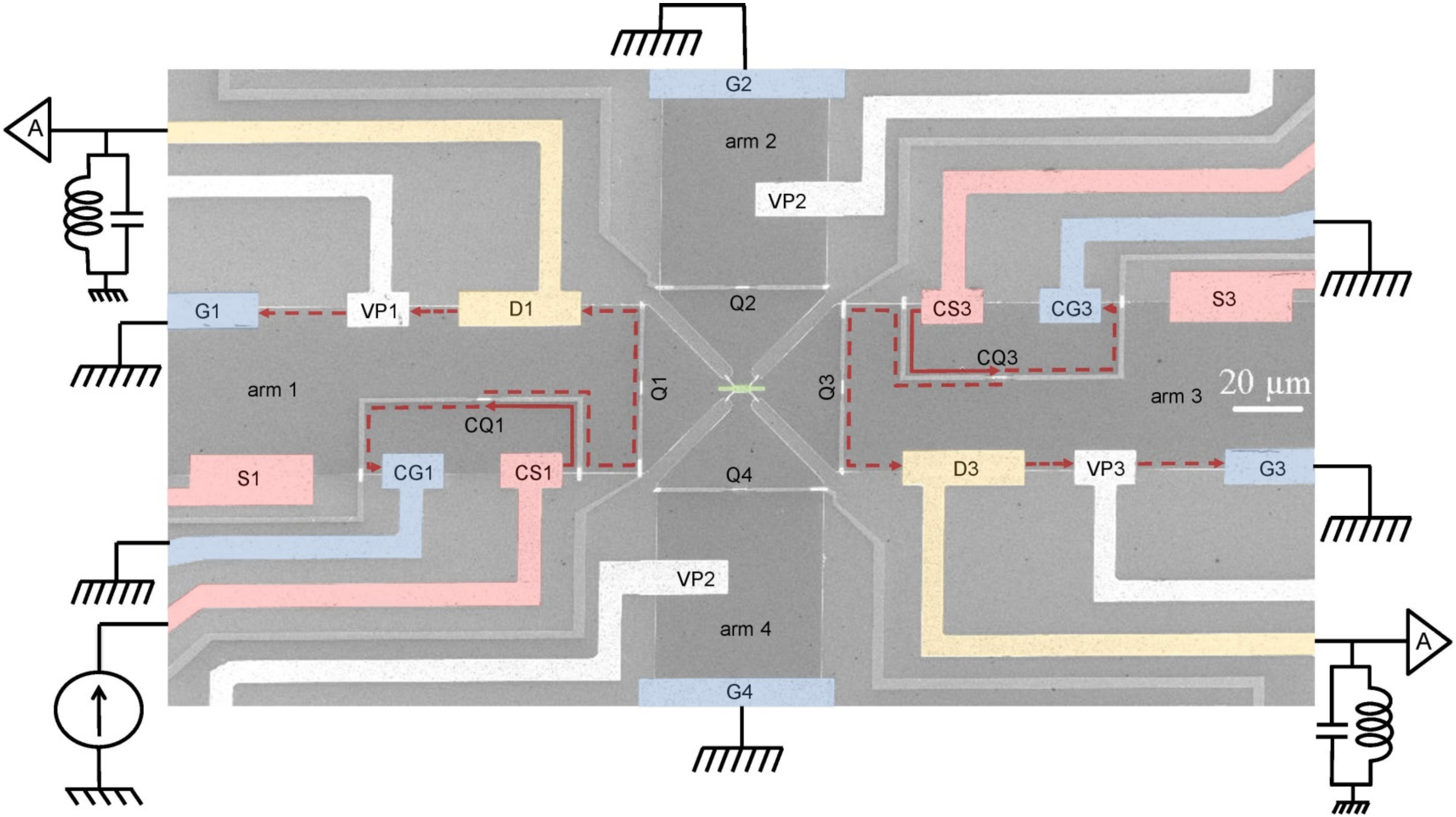


heart of structure

$N = 2 \quad \nu = 2 \quad \nu_{QPC} = 1$



typical actual structure



points of consideration *not an easy experiment*

- electrons fully equilibrate in the small floating reservoir T_m
- outgoing charge channels carry **only** Johnson-Nyquist noise

  without shot noise
- no presence of bulk energy modes (may increase the *apparent* thermal conductance)
- length of arms is limited (~150μm, temperature equilibration between up-down modes)
- equal splitting between arms, amplifier gain determination, contacts' resistance, ...

integers…………………… $\nu = 1, 2$

lowest Landau level

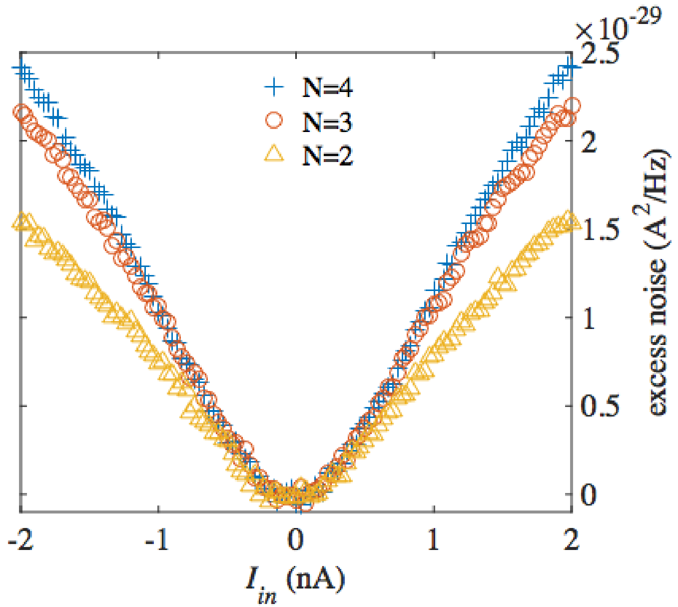
particle-like fractions…… $\nu = 1/3$

hole-like fractions……… $\nu = 2/3, \; 3/5, \; 4/7$

first excited Landau level

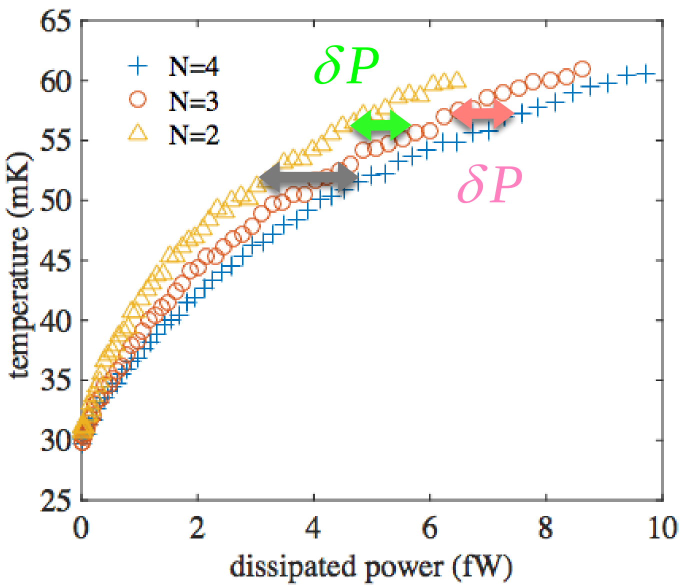
$\nu = 7/3, \; 5/2, \; 8/3$

$\nu = 2$



×10^-29
N=4
N=3
N=2
excess noise (A^2/Hz)
I_{in} (nA)

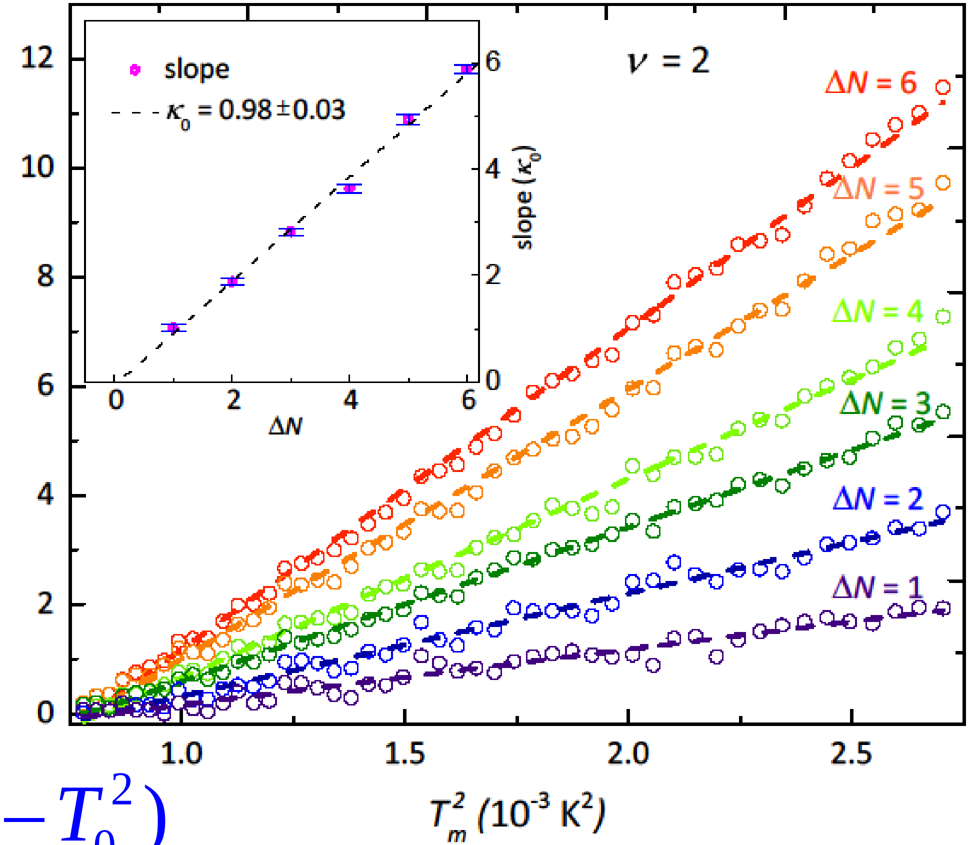
T_0=30mK



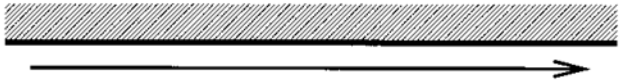
N=4
N=3
N=2
δP
δP
temperature (mK)
dissipated power (fW)

$\nu = 2$

$\delta P_{\Delta N} / 0.5\kappa_0 \ (10^{-3} K^2)$

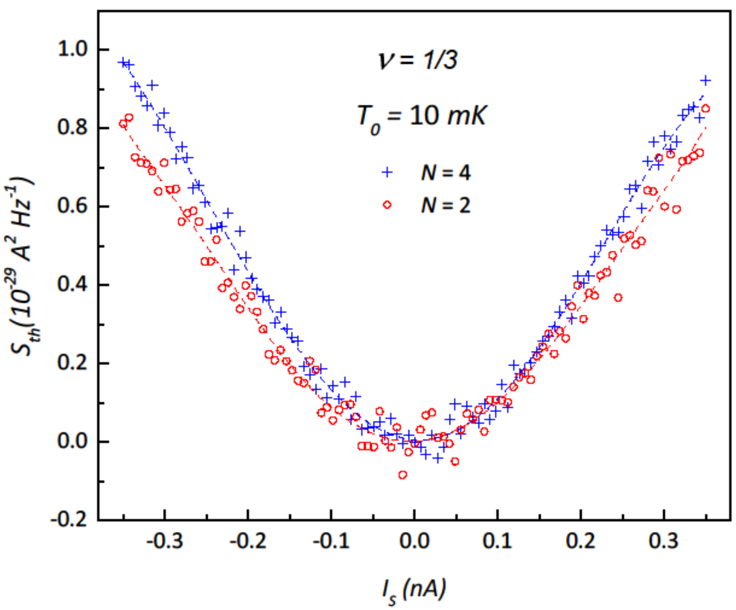


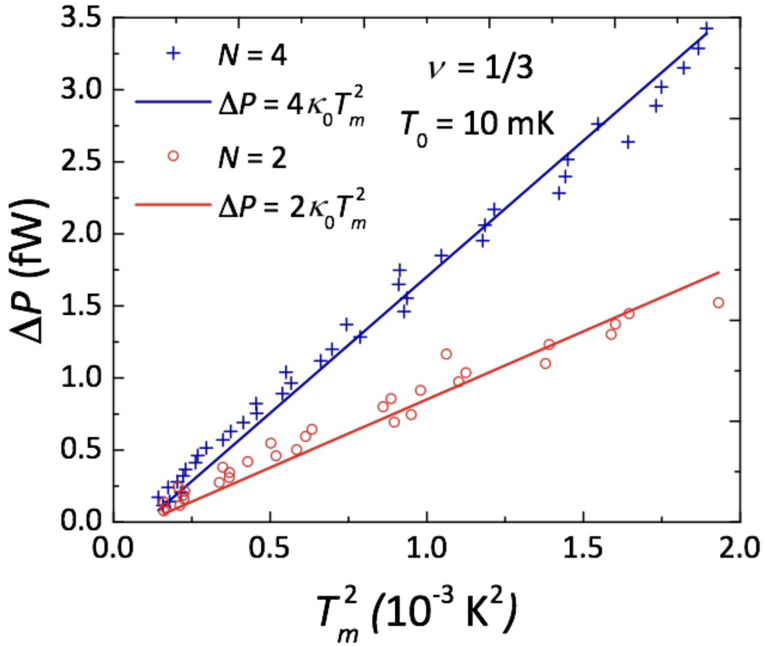
$$J_e \cong 1 \cdot 0.5\kappa_0 (T_m^2 - T_0^2)$$



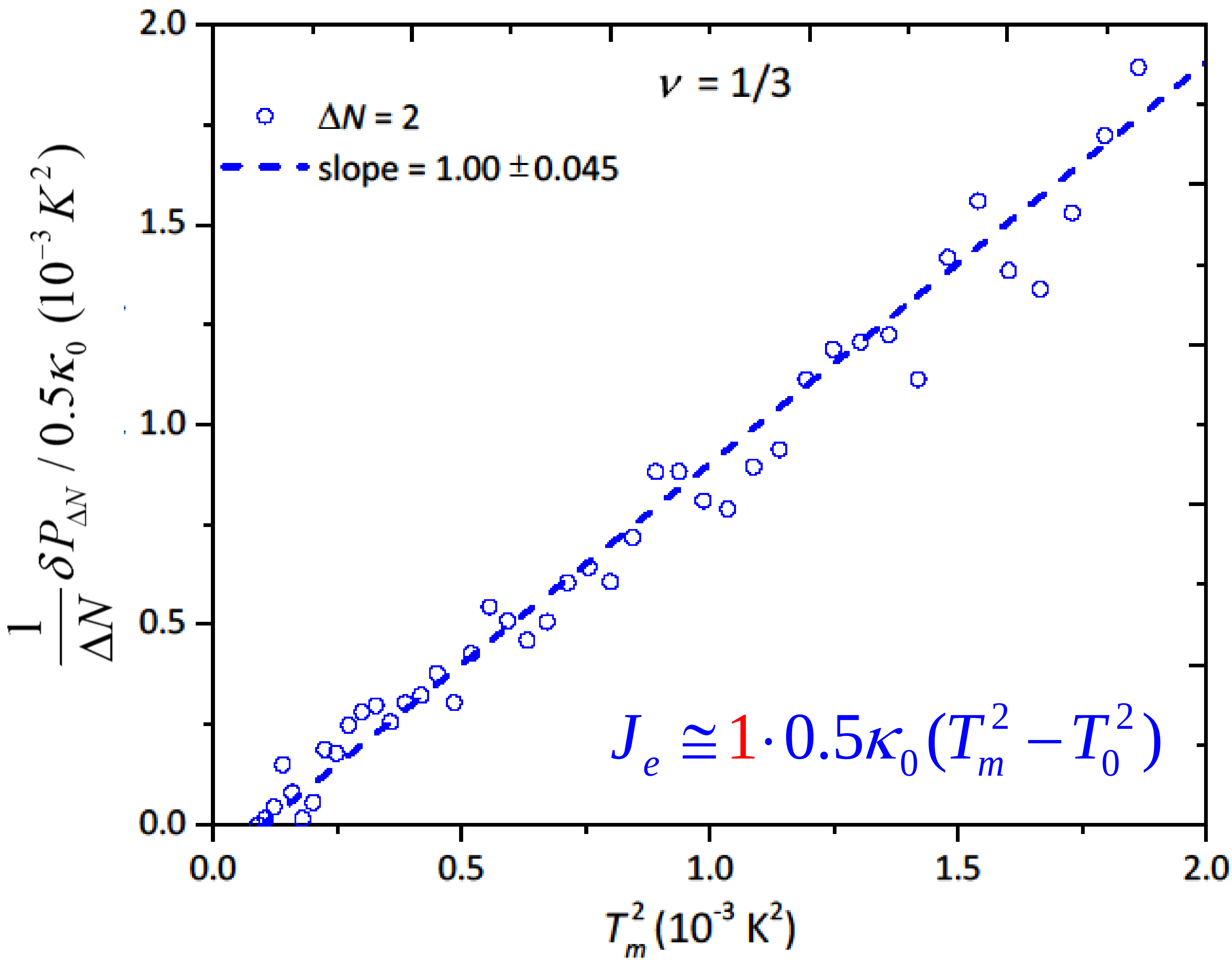
$\nu = 1/3 \rightarrow K = \kappa_0$

ν = 1/3





no phonon contribution



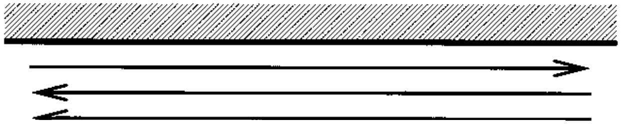
ν = 1/3
ΔN = 2
slope = 1.00 ± 0.045
$J_e \cong 1 \cdot 0.5\kappa_0 (T_m^2 - T_0^2)$
$\frac{1}{\Delta N}\delta P_{\Delta N} / 0.5\kappa_0 \, (10^{-3} K^2)$
$T_m^2 \, (10^{-3} K^2)$
0.0
0.5
1.0
1.5
2.0

κ of hole - states…*Kane & Fisher* 1997

predicted….'bulk-edge' correspondence

$\nu = 2/3 \rightarrow \kappa = 0$

1 charge down + **1** neutral up



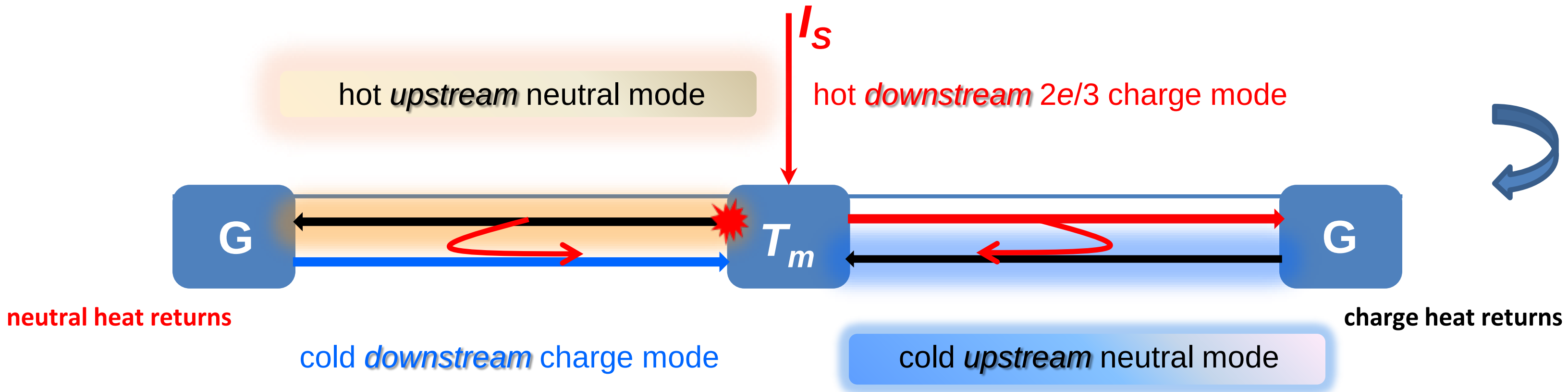
$\nu = 3/5 \rightarrow -\kappa_0$

1 charge down + **2** neutral up

$\nu = 4/7 \rightarrow -2\kappa_0$

1 down charge + **3** neutrals up

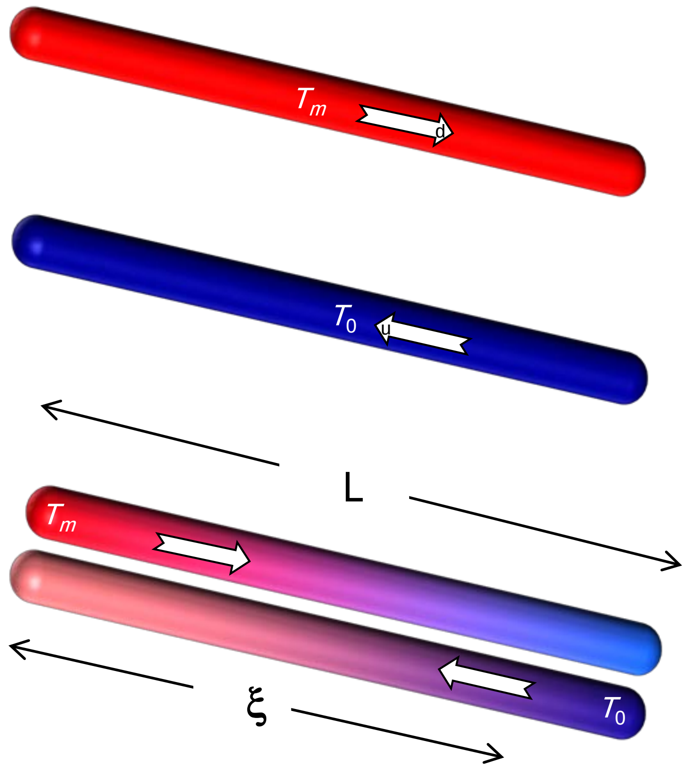
$\nu = 2/3$ …… why $K = 0$?



<u>equal</u> number of **down** and **up** modes

full equilibration ONLY at large length…..**all** emitted heat **returns**

$\nu = 2/3$



non-interacting

interacting

long equilibration length, ξ

modes do not equilibrate

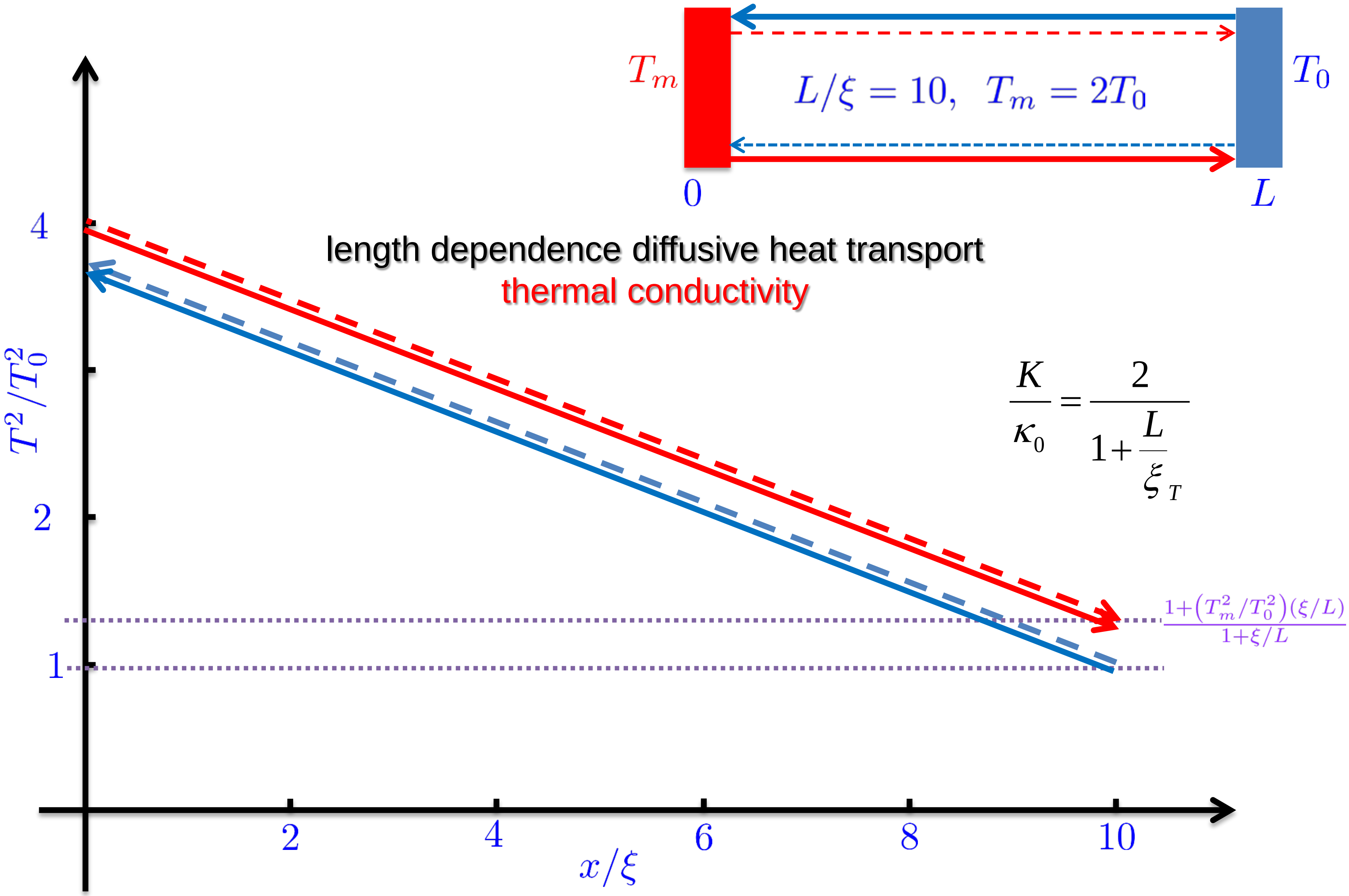
equal number of **down** and **up** modes

heat diffuses in ν =2/3

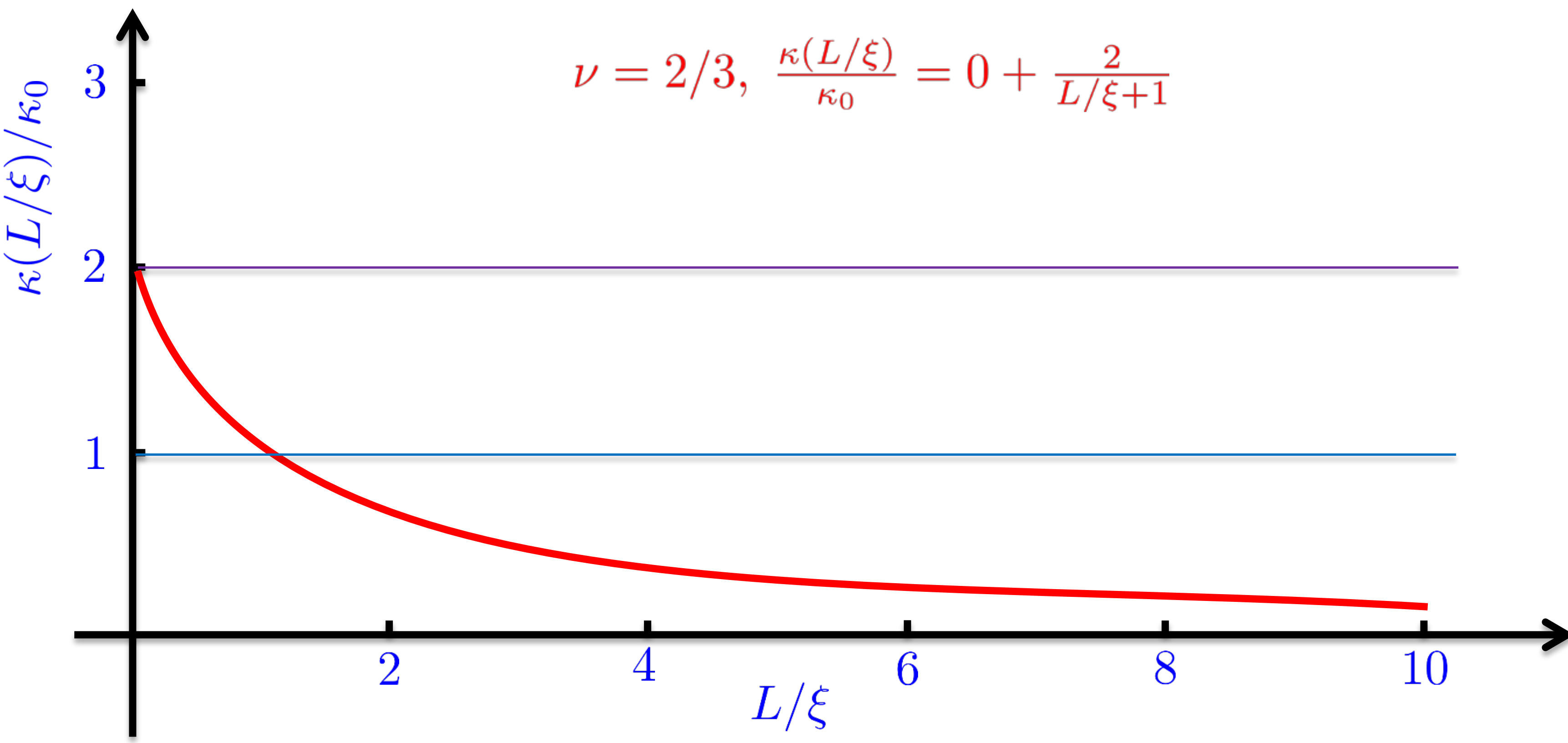
length dependence thermal conductance

‘thermal conductivity’

temperature profile, $\nu = 2/3$

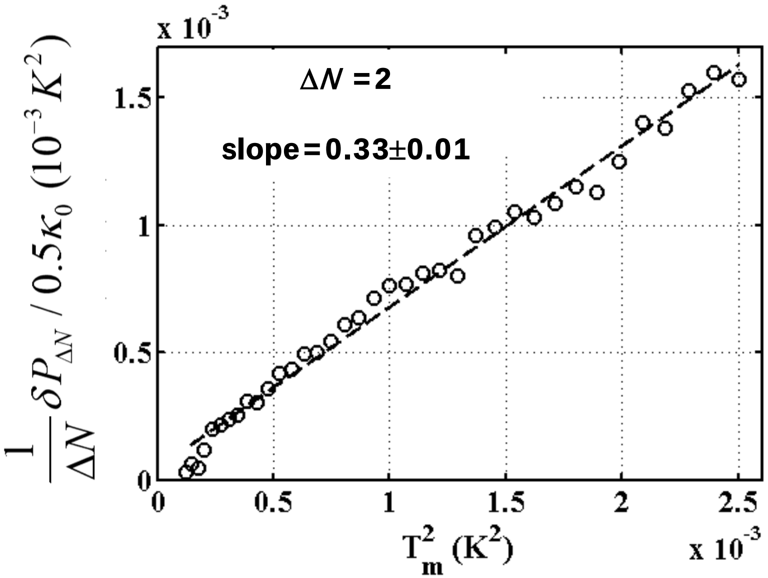


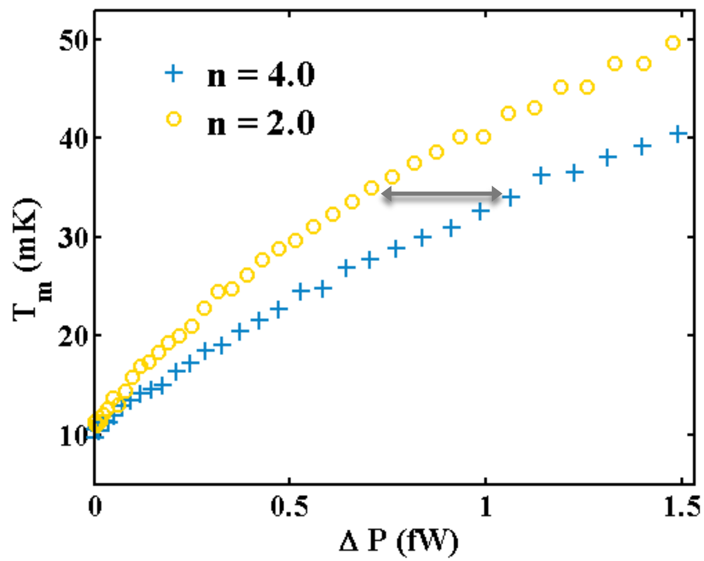
heat conductance w/length



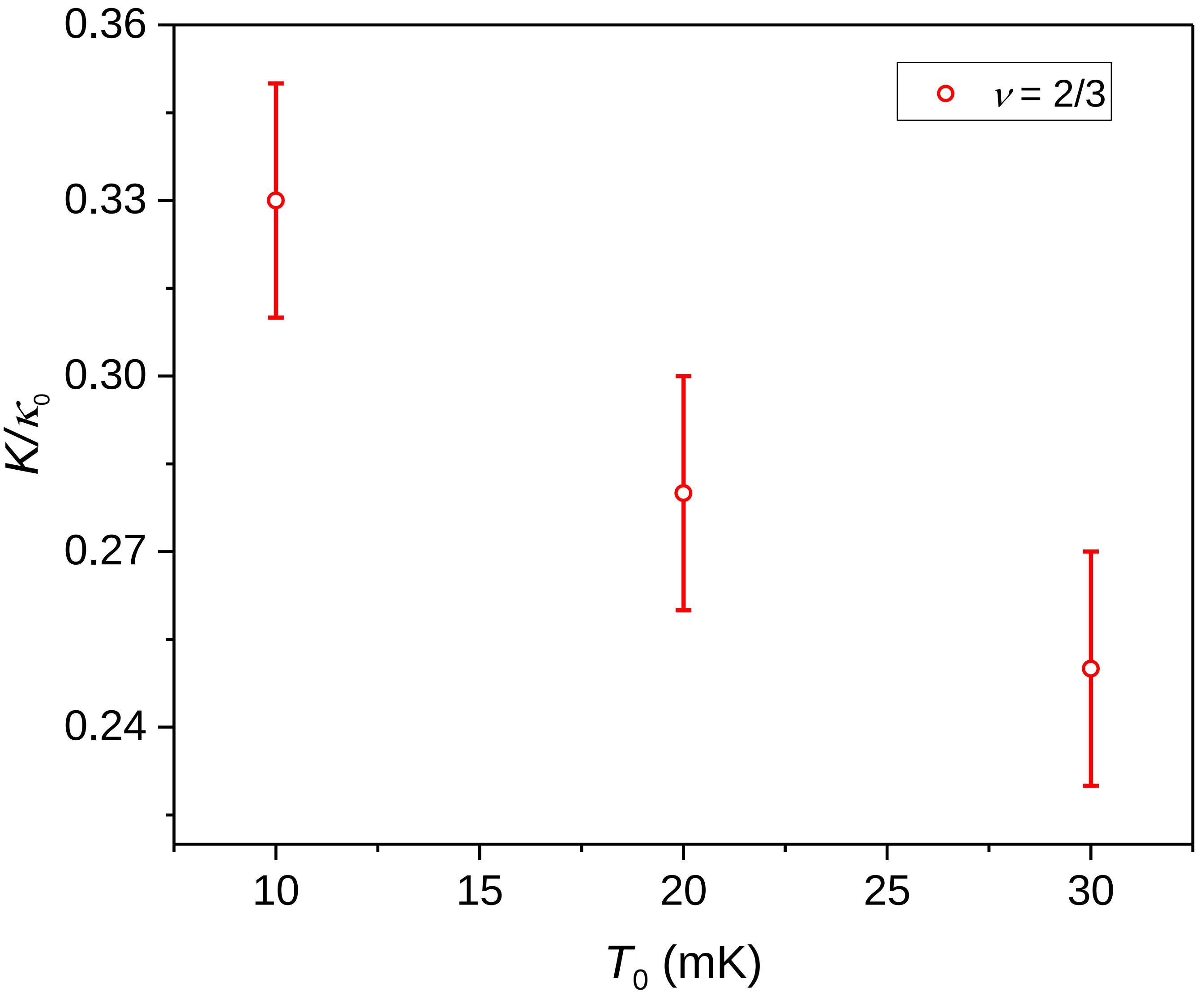
$\nu = 2/3$

$$J_e \cong 0.33 \cdot 0.5\kappa_0 (T_m^2 - T_0^2) \quad T_0 = 10mK$$





$K > 0$…..symmetric up and down of arms……. **$K/2$** each side of arm



0.36
0.33
0.30
0.27
0.24
10
15
20
25
30
K/κ0
T0 (mK)
ν = 2/3

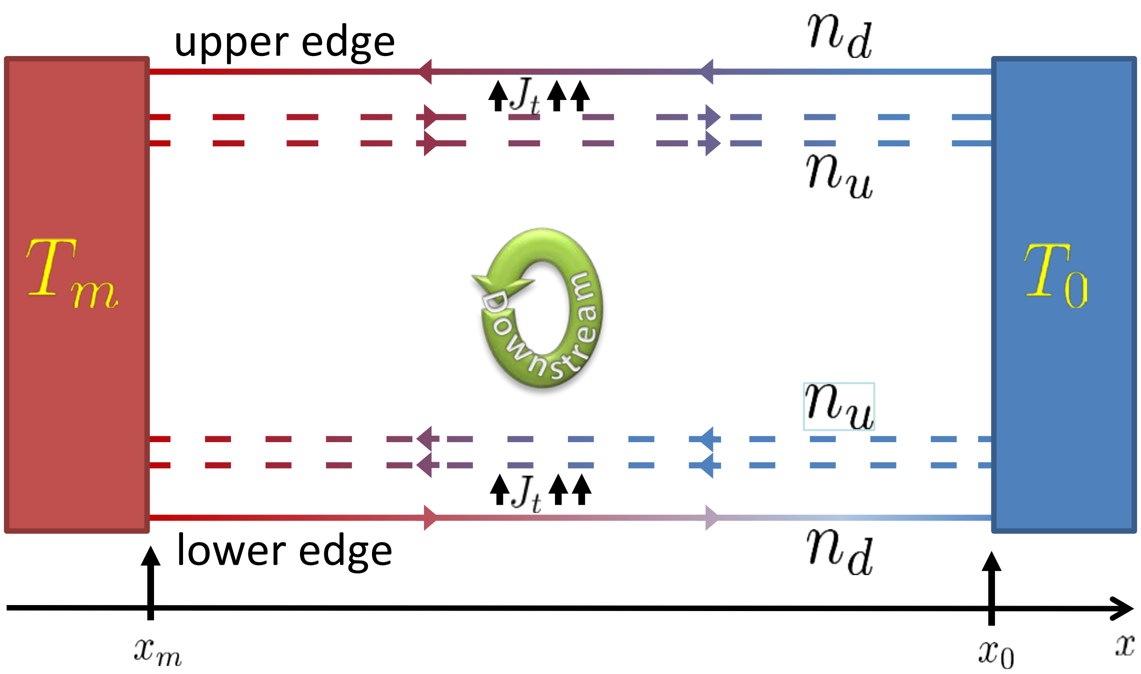
$\nu = 2/3$

$$\frac{K}{\kappa_0} = \frac{2}{1 + \dfrac{L}{\xi_T}} \qquad L \sim 150\,\mu m$$

$$T_m^{ava} = 20mK \qquad \xi_T = 30\,\mu m \qquad J_e \simeq 0.33 \cdot 0.5\kappa_0 (T_m^2 - T_0^2)$$

$$T_m^{ava} = 45mK \qquad \xi_T = 20\,\mu m \qquad J_e \simeq 0.25 \cdot 0.5\kappa_0 (T_m^2 - T_0^2)$$

calculating T(x) & K … … v =3/5



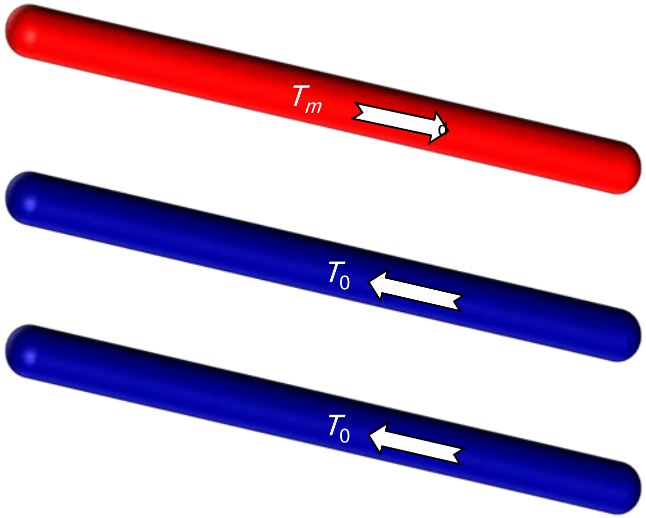
$n_d=1$ $n_u=2$

$$J = KT^2$$

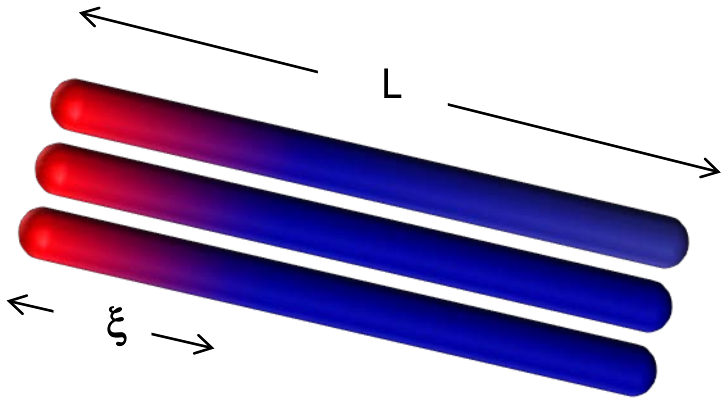
$$0.5 n_u \kappa_0 \partial_x T_u^2(x) = -j_t(x)$$

$$0.5 n_d \kappa_0 \partial_x T_d^2(x) = -j_t(x)$$

$\nu = 3/5$



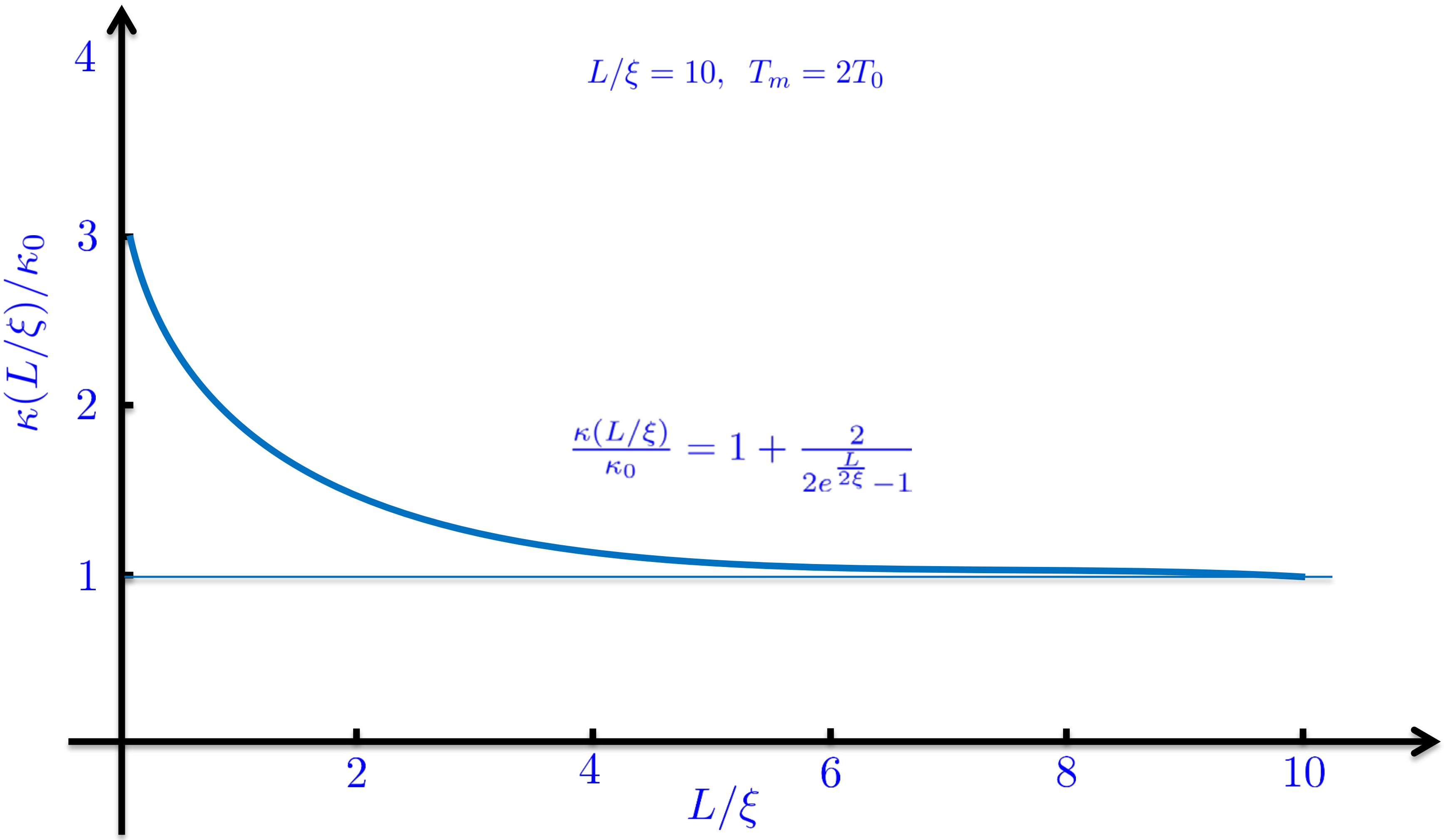
unequal number of **down** and **up** modes

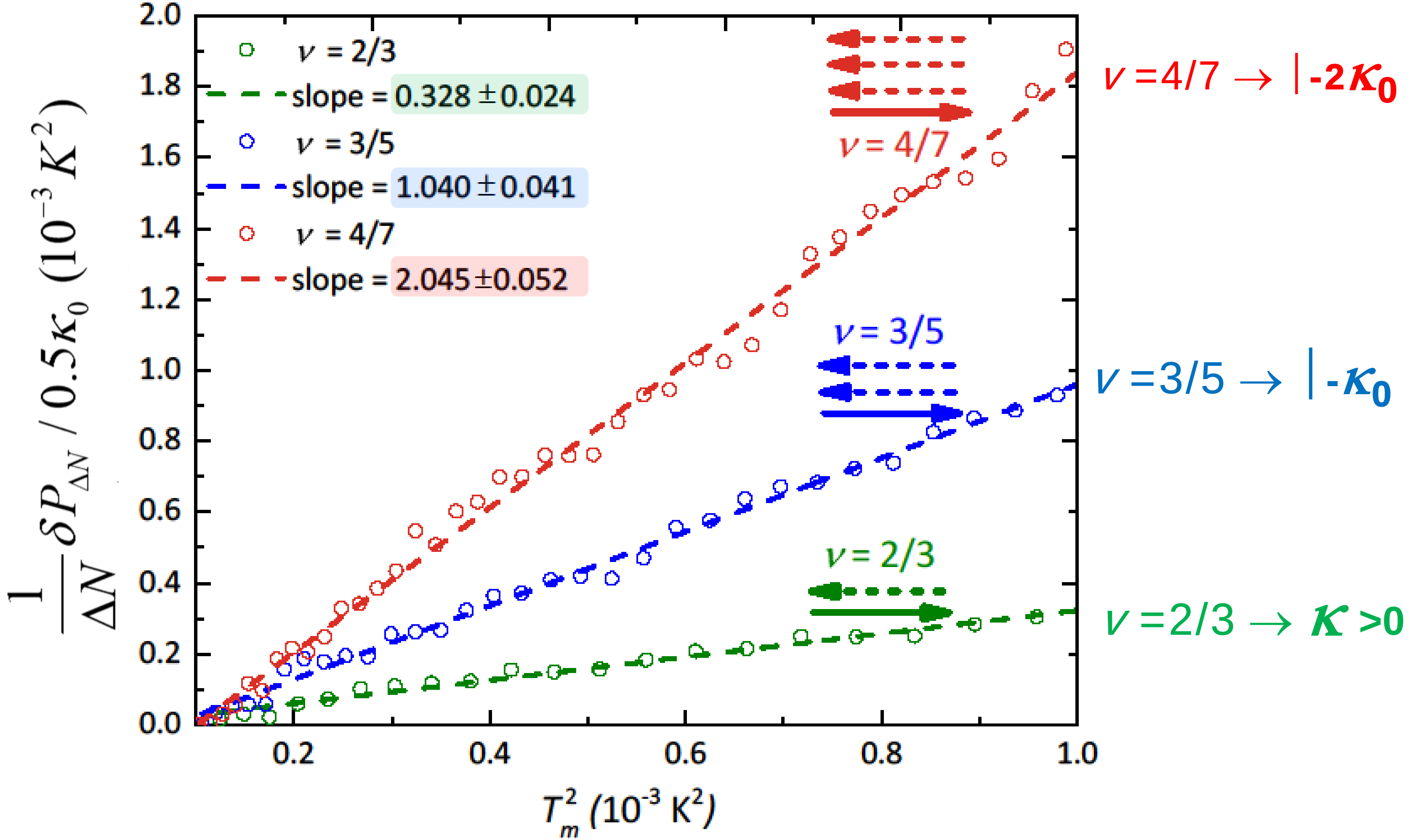


shorter ξ

nodes equilibrate

heat conductance w/length $\nu = 3/5$





hole-states with upstream neutral modes

fractional interacting 1D mode

and neutral mode

$K = \kappa_0$

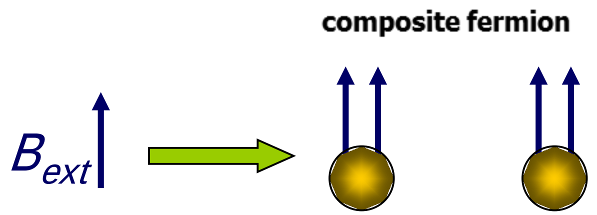
second Landau level

already known for $\nu = 5/2 = 2 + 1/2$

- charge $e/4$
- upstream neutral modes
- likely, spin polarized

Moore - Read theory 1991

ν=5/2 = 2+1/2

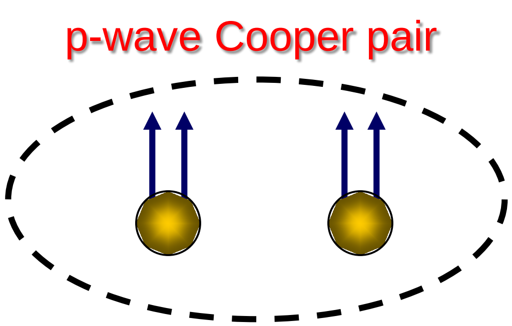


$\Delta B = 0$

R_{xx}=0 ...superconductor

BCS of ***polarized*** *composite fermions*

w/ odd orbital angular momentum



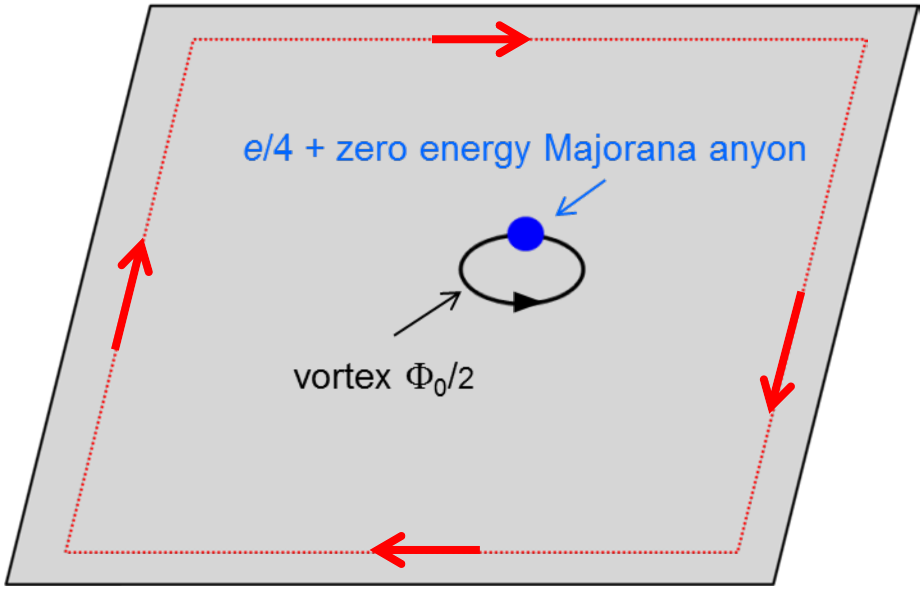
zero energy Majorana anyons

- $B - B_{1/2} > 0$ induces vortices in bulk
- zero energy quasiparticle (Majorana) in vortex + $e/4$
- Majorana's come in pairs.....forming fermionic state $\gamma_1 \pm i\gamma_2$
- ground state degeneracy of n vortices....... $2^{n/2}$ (non-abelian)

$$\begin{cases} \Gamma_1 = \Gamma_1^\dagger \\ \Gamma_2 = \Gamma_2^\dagger \end{cases} \qquad \begin{cases} a = \frac{1}{2}(\Gamma_1 + i\Gamma_2) \\ a^\dagger = \frac{1}{2}(\Gamma_1 - i\Gamma_2) \end{cases}$$

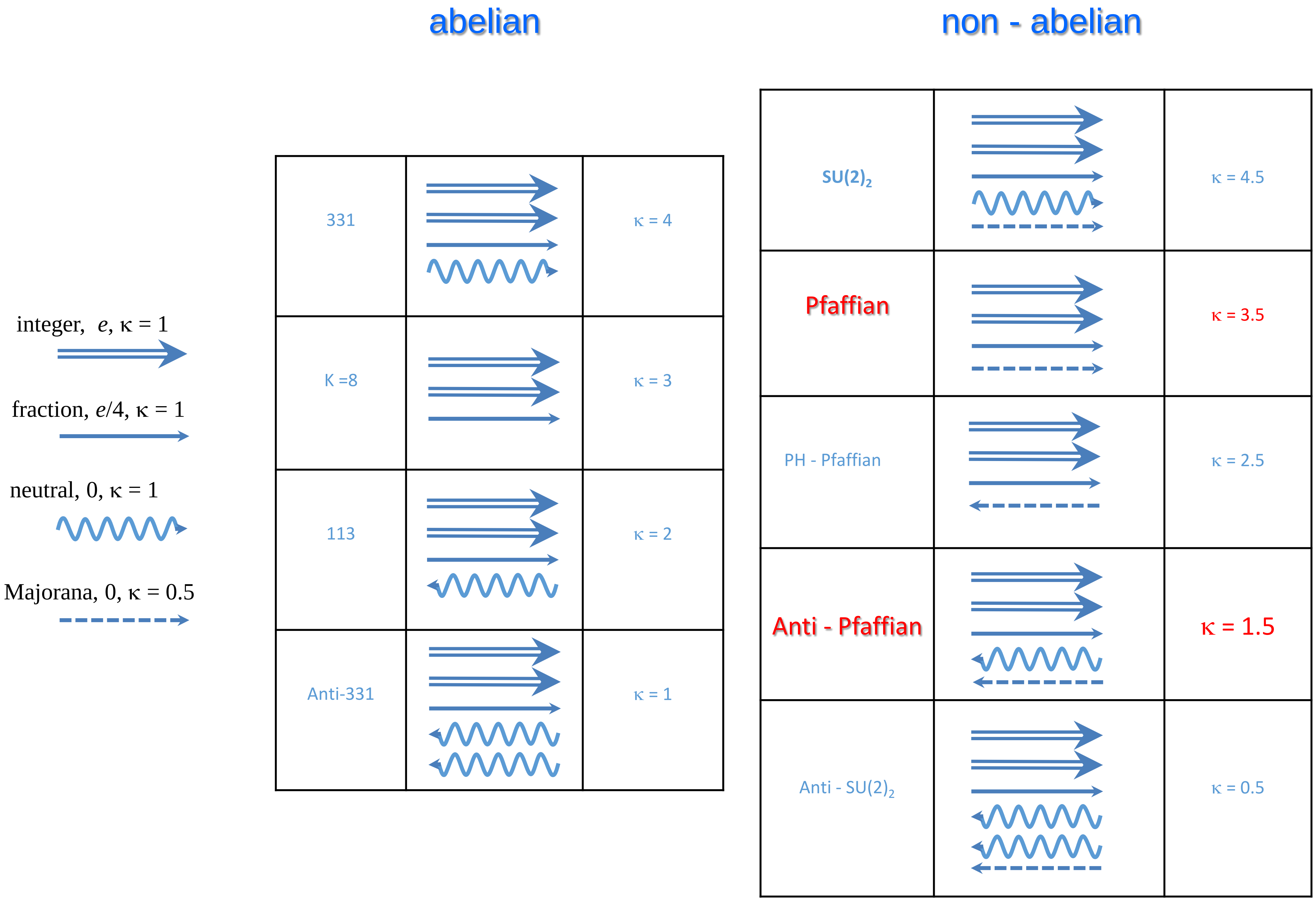
5/2 state   Moore – Read, Pfaffian state

bulk – edge correspondence



1D edge liquid + Majorana anyon

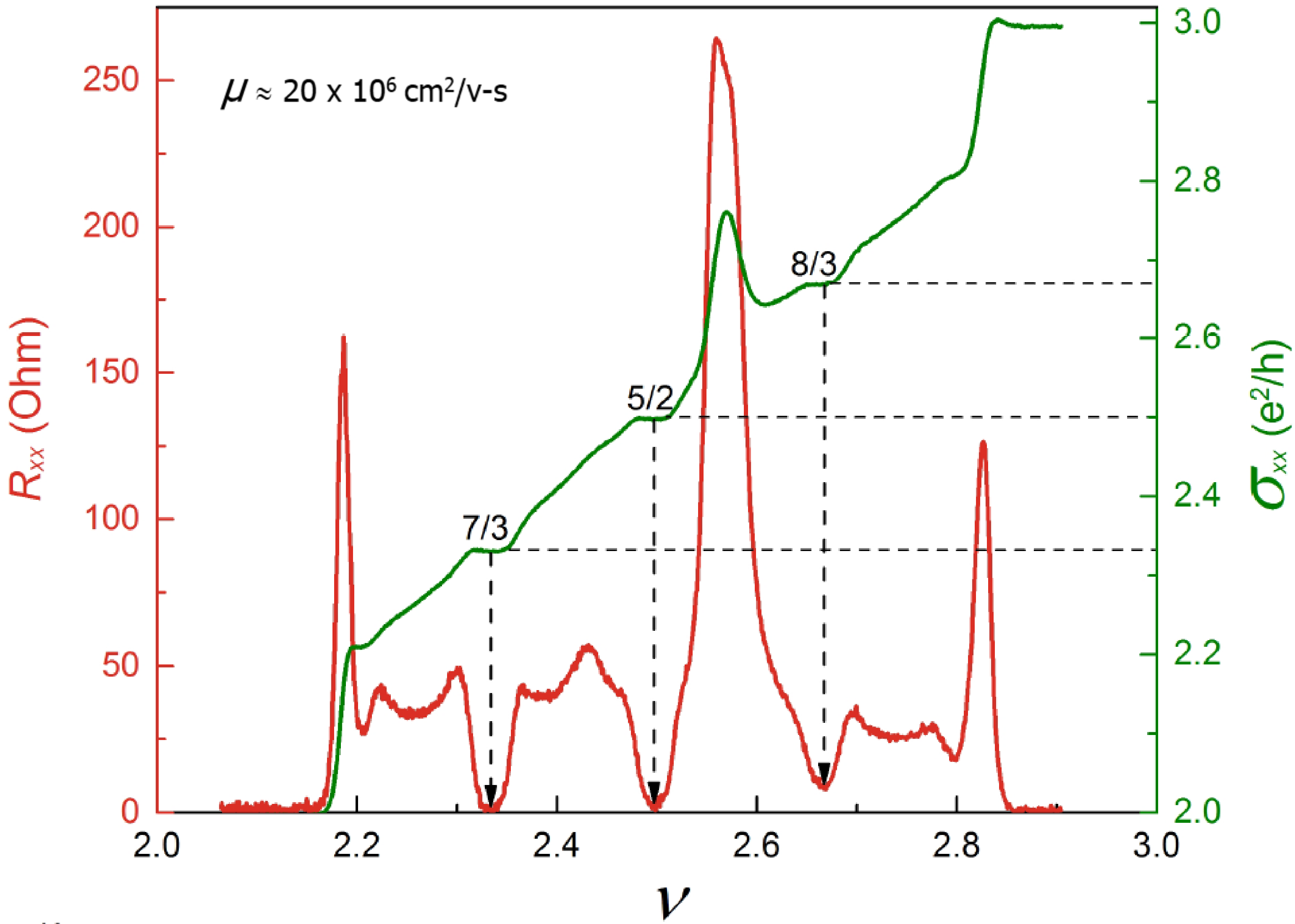
Majorana – half fermion…. $K = \kappa_0 / 2$


abelian
non - abelian
integer, e, κ = 1
fraction, e/4, κ = 1
neutral, 0, κ = 1
Majorana, 0, κ = 0.5
331
κ = 4
K =8
κ = 3
113
κ = 2
Anti-331
κ = 1
SU(2)2
κ = 4.5
Pfaffian
κ = 3.5
PH - Pfaffian
κ = 2.5
Anti - Pfaffian
κ = 1.5
Anti - SU(2)2
κ = 0.5

difficulties in 5/2 material

- ‘bulk heat conductance’…..free electrons in the donor layers
- poor contact of floating reservoir – reflections of inner modes
- instability and hysteresis of QPC’s

non-standard MBE growth for ν =5/2



neglibable bulk thermal conductance

$\nu = 7/3$ $\nu = 2 + 1/3$ particle like, downstream

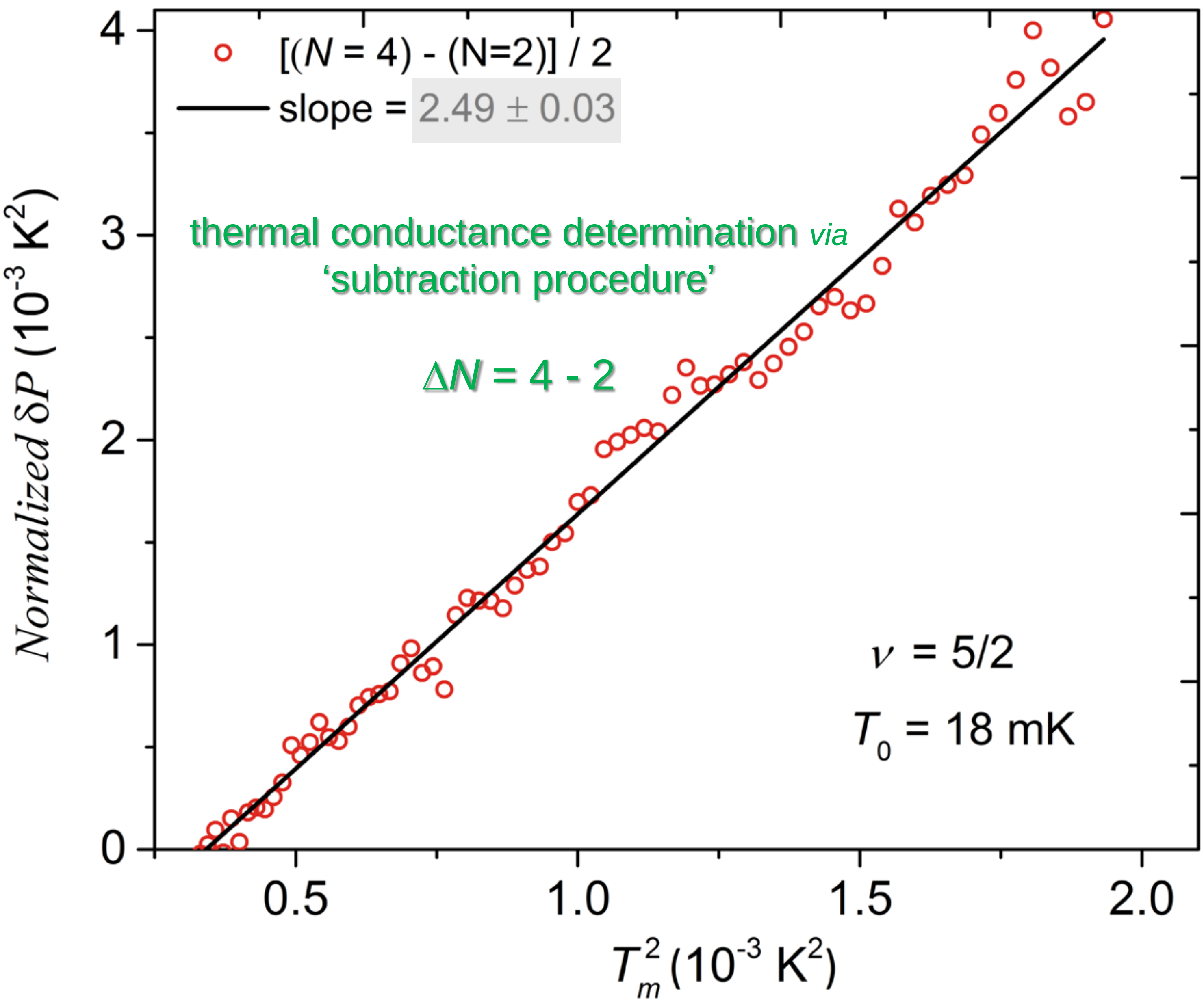
$\nu = 8/3$ $\nu = 2 + 2/3$ hole-like, down - up

measured

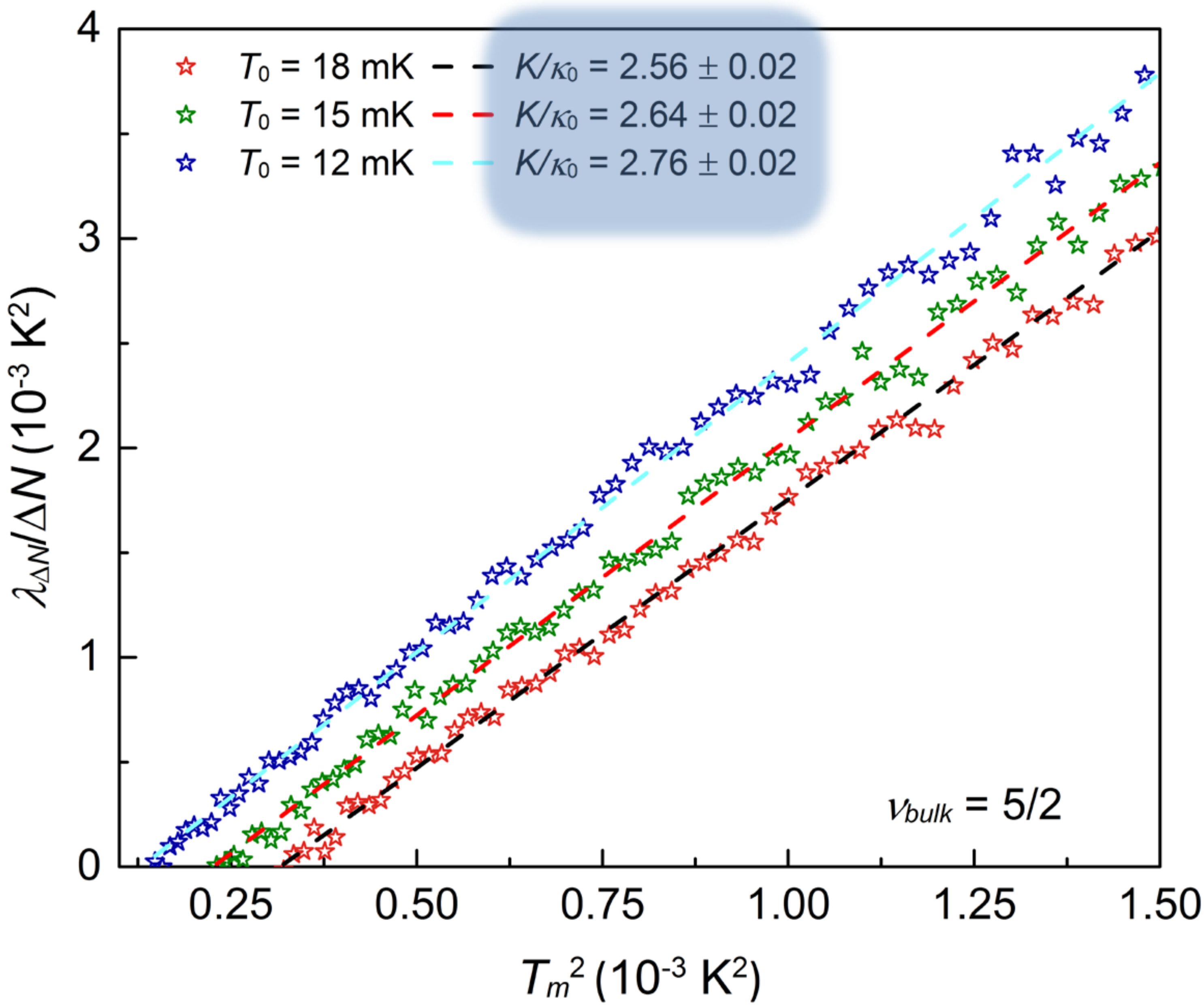
$K = 3\kappa_0$

$K = (2+\varepsilon)\kappa_0$

$\nu = 5/2$ $\nu = 2 + \frac{1}{2}$



[(N = 4) - (N=2)] / 2
slope = 2.49 ± 0.03
thermal conductance determination via 'subtraction procedure'
ΔN = 4 - 2
ν = 5/2
T0 = 18 mK
Normalized δP (10-3 K2)
Tm2 (10-3 K2)
0
1
2
3
4
0.5
1.0
1.5
2.0



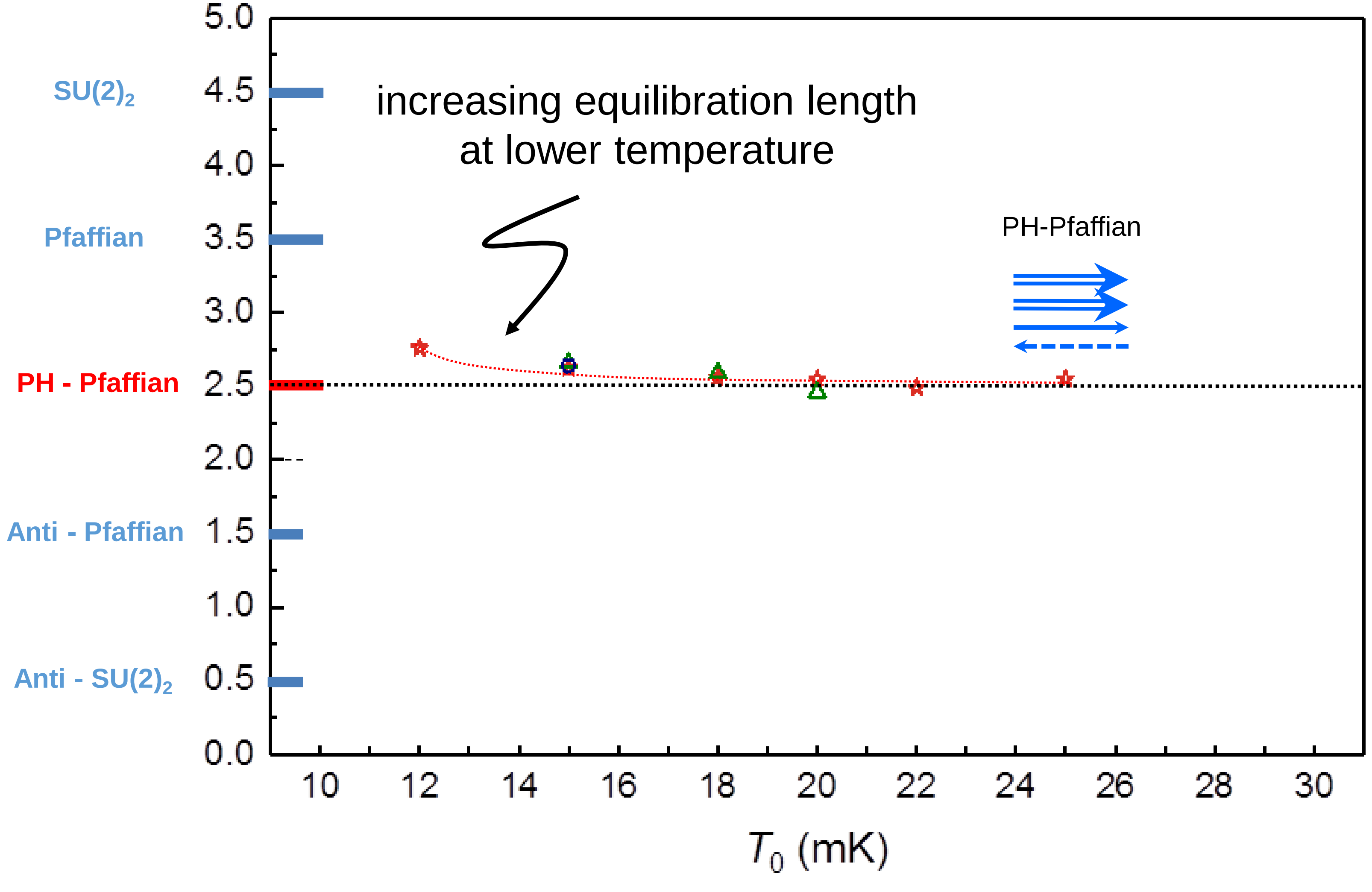
T_0 = 18 mK
T_0 = 15 mK
T_0 = 12 mK
K/κ_0 = 2.56 ± 0.02
K/κ_0 = 2.64 ± 0.02
K/κ_0 = 2.76 ± 0.02
$\lambda_{\Delta N}/\Delta N$ (10^{-3} K^2)
T_m^2 (10^{-3} K^2)
ν_{bulk} = 5/2
0
1
2
3
4
0.25
0.50
0.75
1.00
1.25
1.50

three thermal cycling

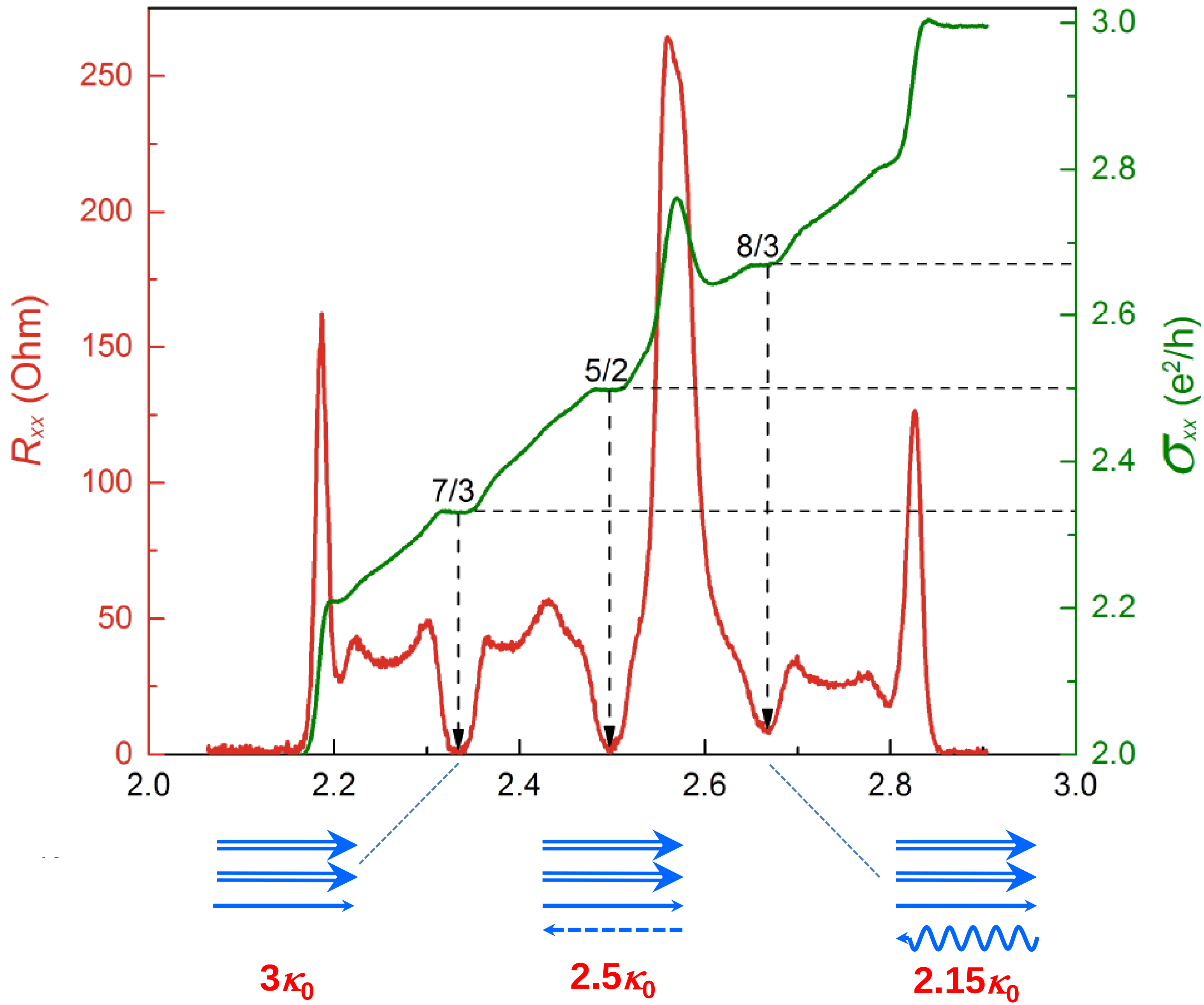
17 measurements

different temperatures

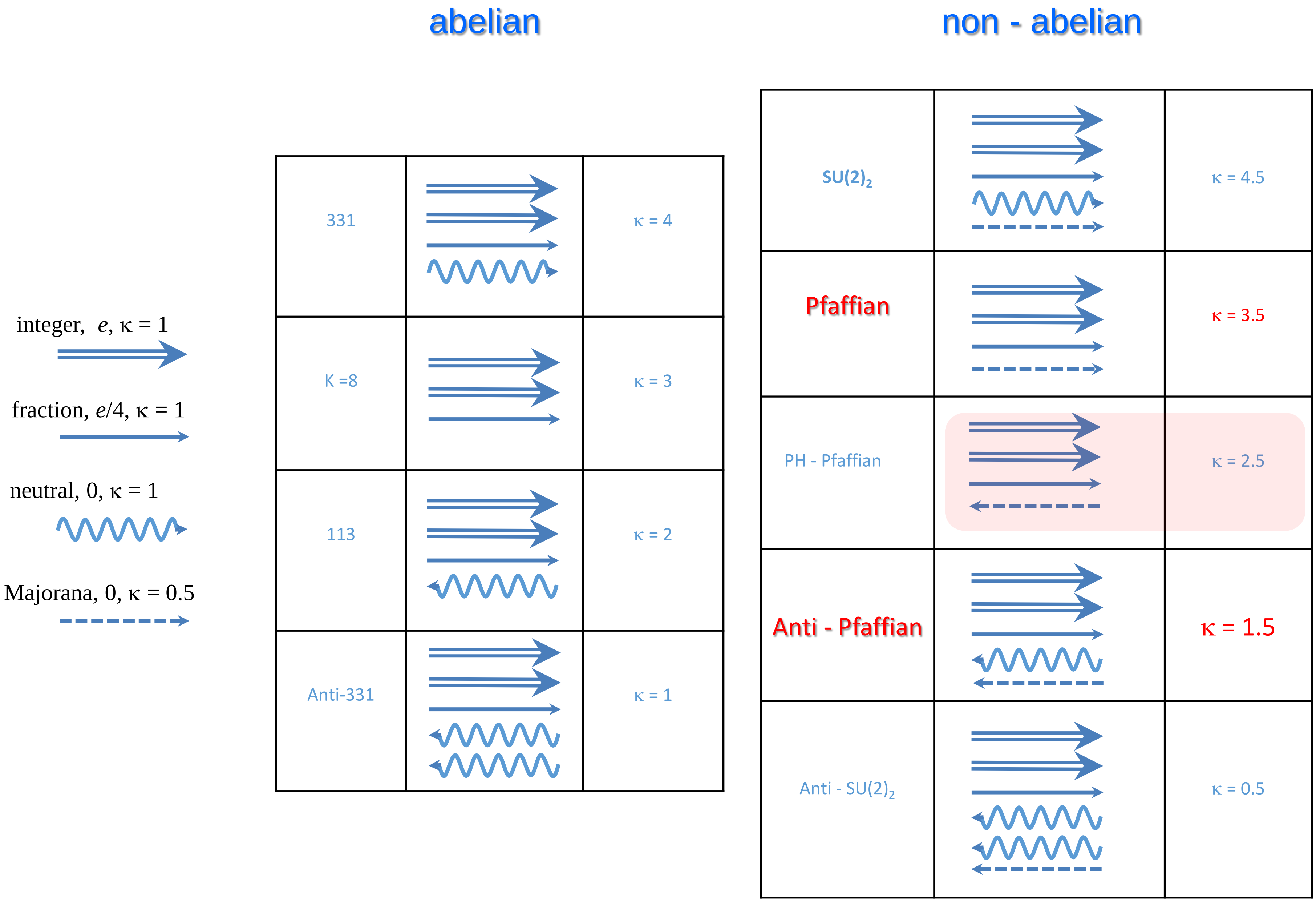
three location of the 5/2 plateau



SU(2)2
Pfaffian
PH - Pfaffian
Anti - Pfaffian
Anti - SU(2)2
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
increasing equilibration length
at lower temperature
PH-Pfaffian
10
12
14
16
18
20
22
24
26
28
30
T0 (mK)



R_{xx} (Ohm)
σ_{xx} (e^2/h)
250
200
150
100
50
0
3.0
2.8
2.6
2.4
2.2
2.0
2.0
2.2
2.4
2.6
2.8
3.0
7/3
5/2
8/3
$3\kappa_0$
$2.5\kappa_0$
$2.15\kappa_0$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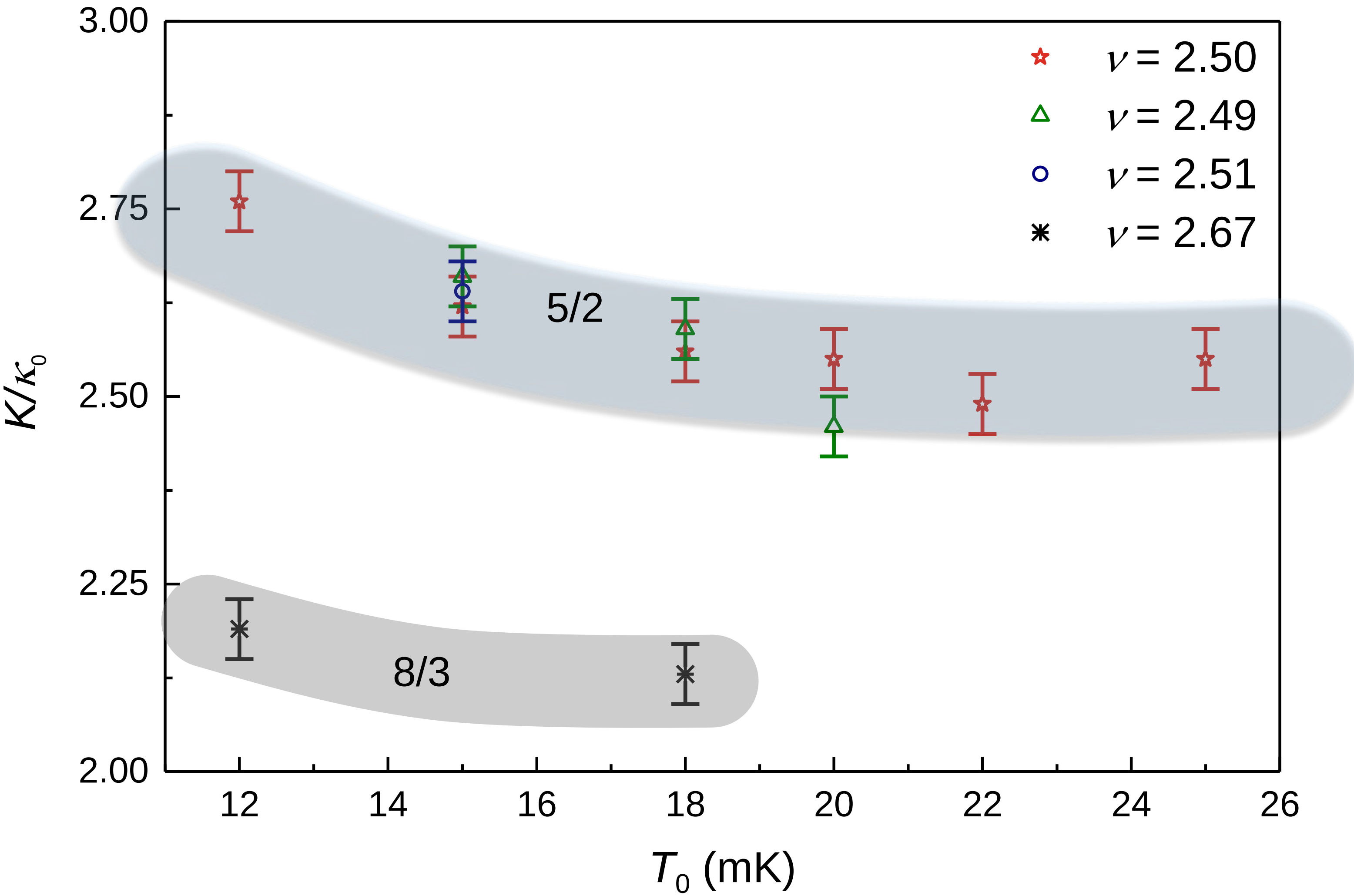
PH - Pfaffian
κ = 2.5
Anti - Pfaffian
κ = 1.5
Anti - SU(2)2
κ = 0.5

ν = 5/2……..non-abelian

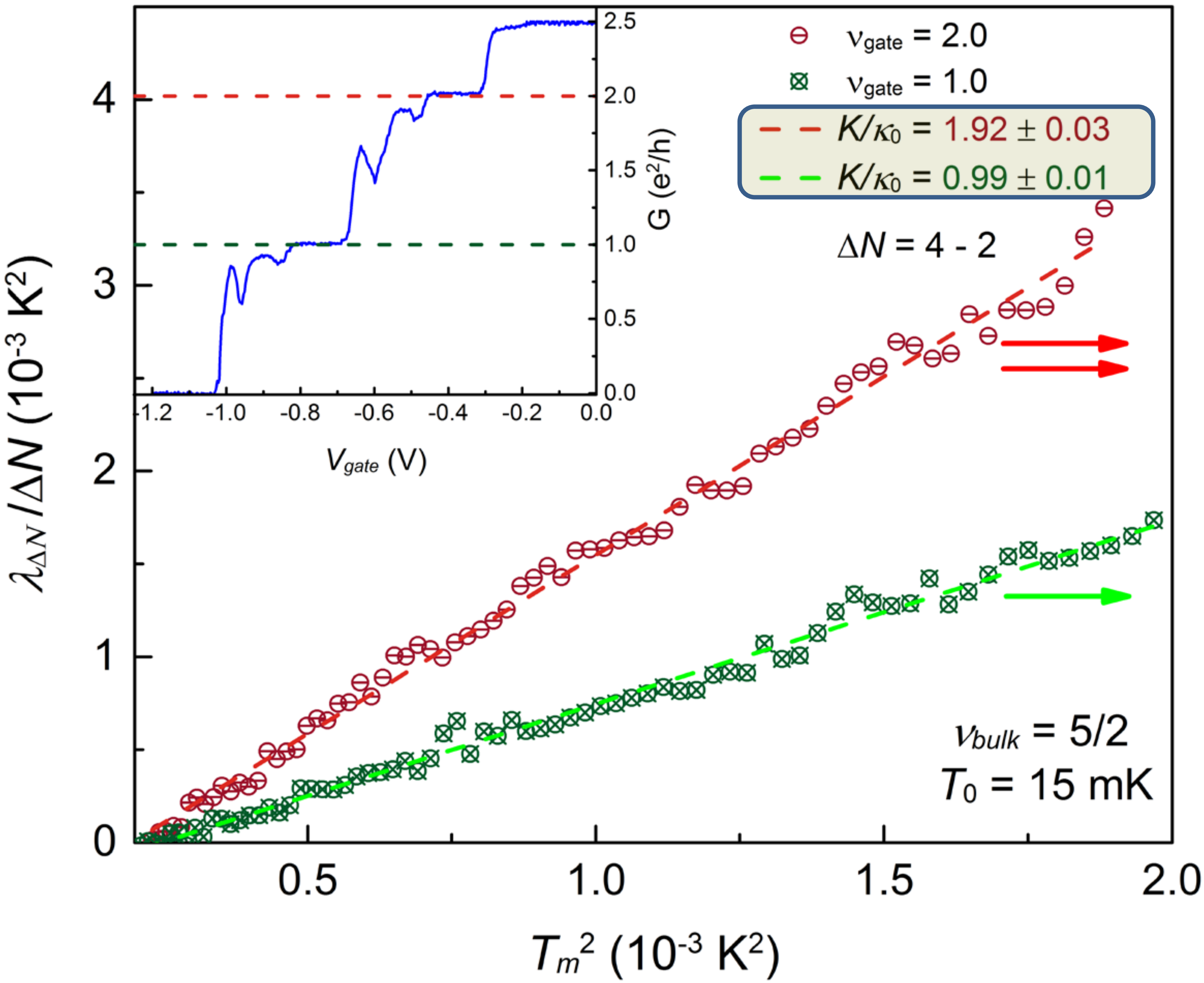
measuring thermal conductance

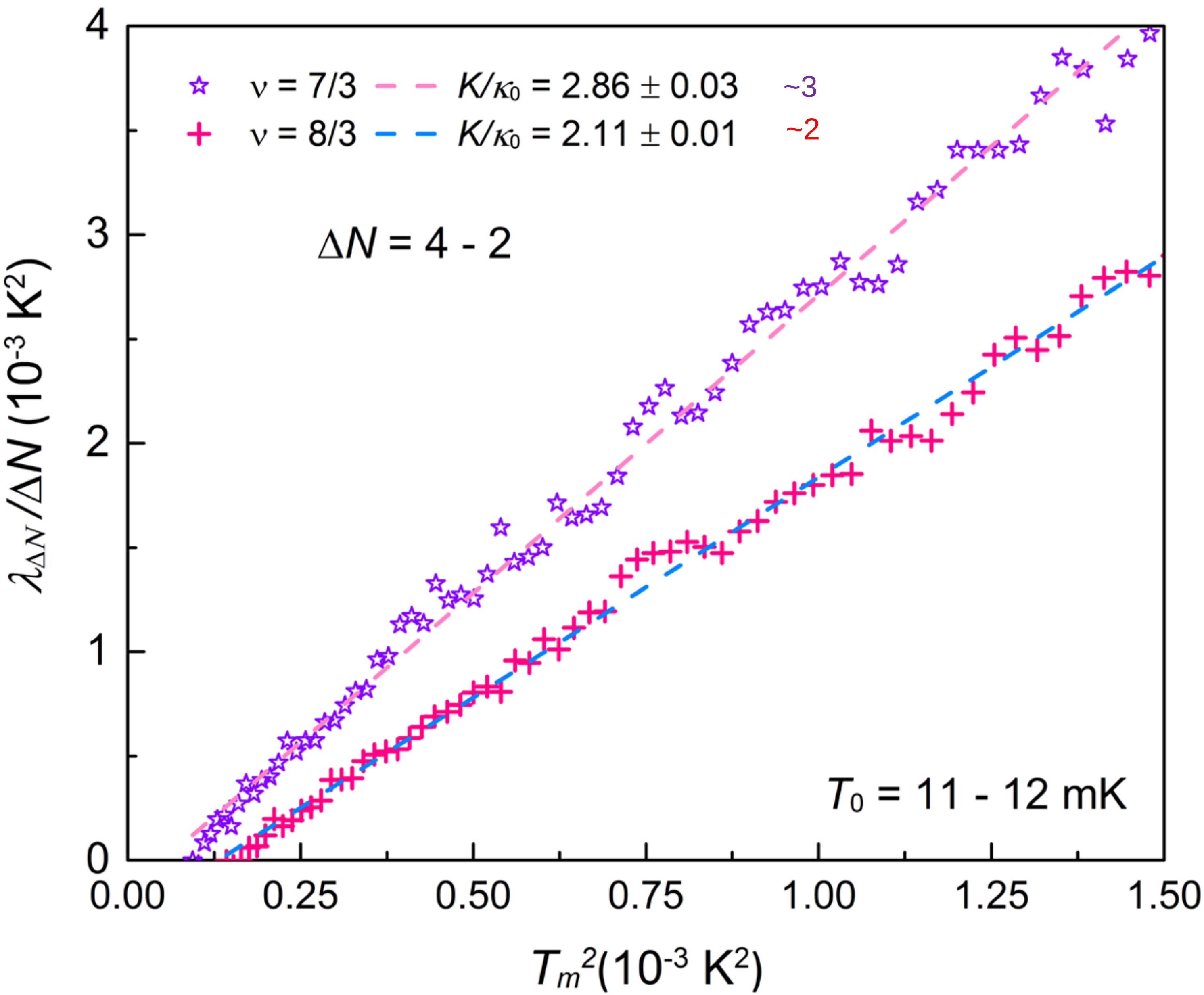
reveals hidden information



3.00
2.75
2.50
2.25
2.00
K/κ_0
12
14
16
18
20
22
24
26
T_0 (mK)
ν = 2.50
ν = 2.49
ν = 2.51
ν = 2.67
5/2
8/3

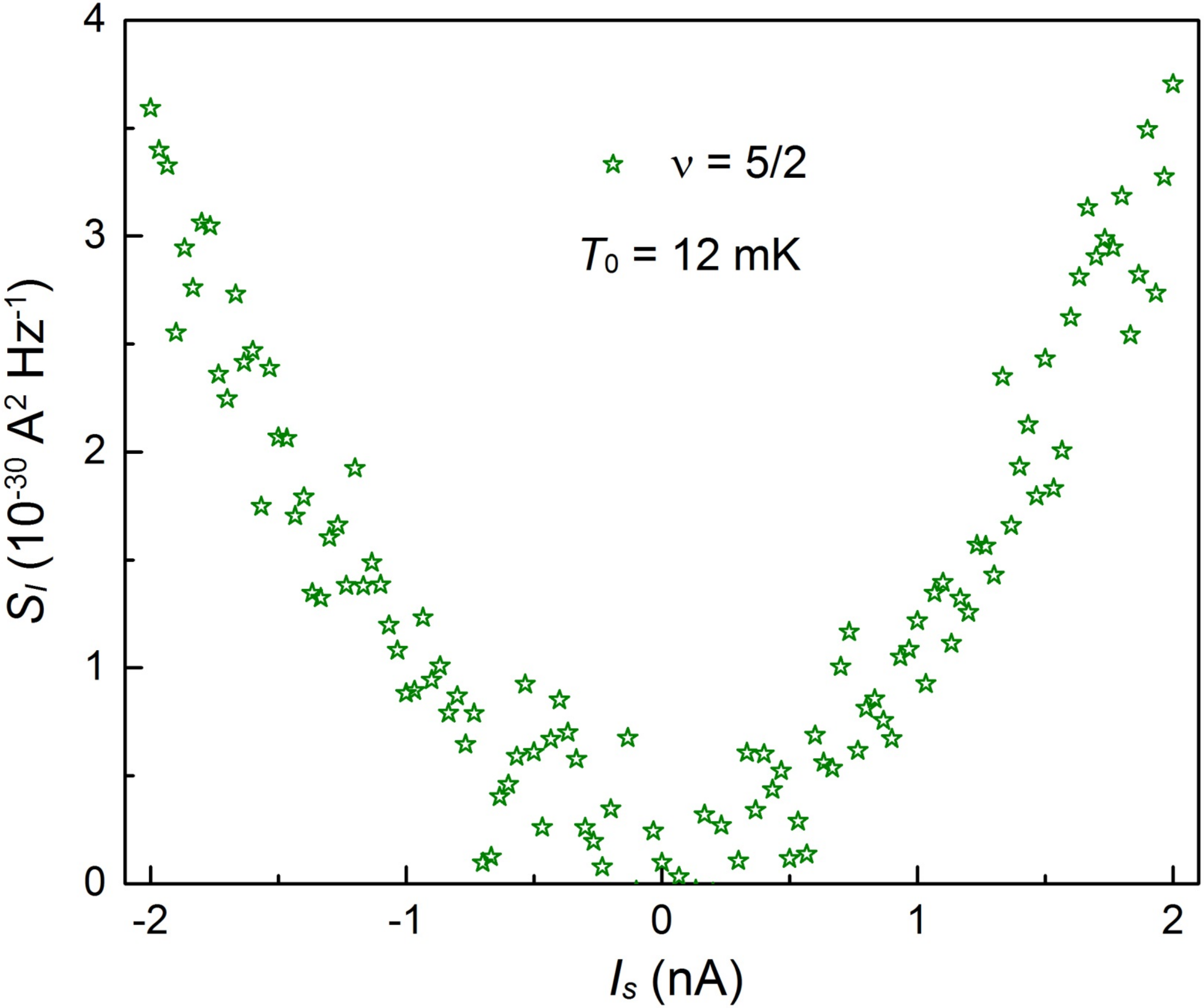
measuring $\nu = 1,2$ @ $\nu_B = 5/2$





ν = 7/3
K/κ0 = 2.86 ± 0.03
~3
ν = 8/3
K/κ0 = 2.11 ± 0.01
~2
ΔN = 4 - 2
T0 = 11 - 12 mK
λΔN/ΔN (10-3 K2)
Tm2(10-3 K2)
0
1
2
3
4
0.00
0.25
0.50
0.75
1.00
1.25
1.50

Neutral Noise



zero energy Majorana anyons

- $B - B_{1/2} > 0$ induces vortices in the p-wave BCS condensate
- zero energy quasiparticle (Majorana) in vortex + $e/4$
- Majorana - chargeless and spinless
- Majorana's come in pairs.....forming fermionic state
- occupied & unoccupied at zero energy
- ground state degeneracy of n vortices....... $2^{n/2}$

$$\begin{cases} \Gamma_1 = \Gamma_1^\dagger \\ \Gamma_2 = \Gamma_2^\dagger \end{cases} \qquad \begin{cases} a = \frac{1}{2}(\Gamma_1 + i\Gamma_2) \\ a^\dagger = \frac{1}{2}(\Gamma_1 - i\Gamma_2) \end{cases}$$