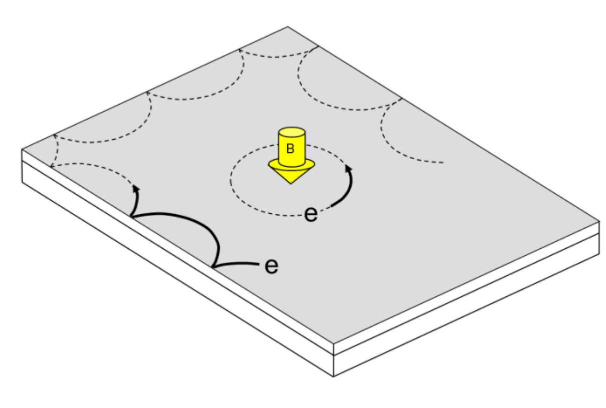
Edge Modes in QHE Regime …. their nature & use

- ‘bulk – edge’ correspondence
- interference
- thermal conductance

Moty Heiblum

WEIZMANN INSTITUTE OF SCIENCE



why 2D electrons.... heart of present transistors

high mobility electrons, gate controlled

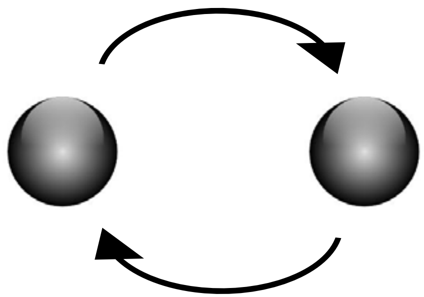
exchange statistics of 2D electrons is rich

exotic states

exchange statistics in 3d

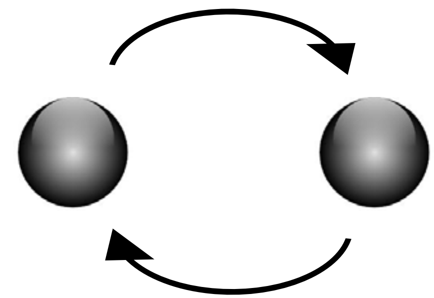
bosons

$\psi \to +\psi$



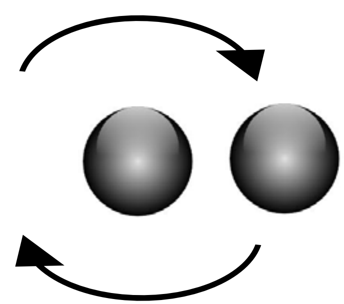
fermions

$\psi \to -\psi$



both

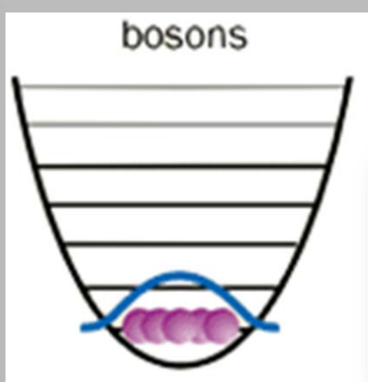
$\psi \to \psi$



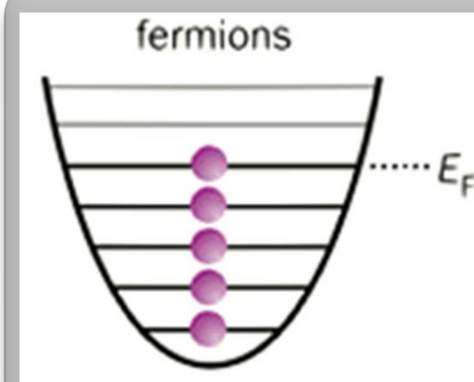
consequences...

bosons

fermions



$$f_{BE} = \frac{1}{e^{(\varepsilon-\mu)/kT} - 1}$$

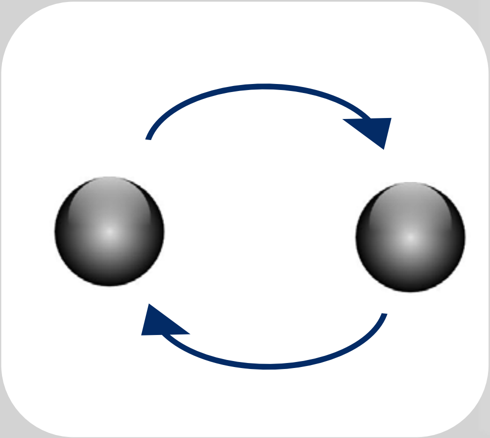


$$f_{FD} = \frac{1}{e^{(\varepsilon-\mu)/kT} + 1}$$

anyonic statistics in 2d → *abelian* (Laughlin qp's)

anyons

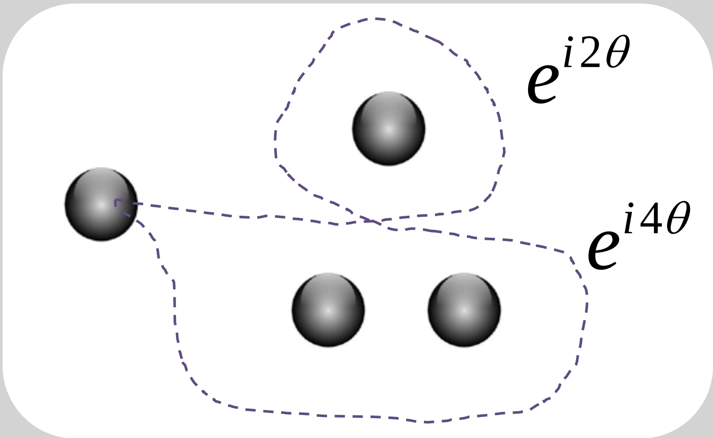
$$\psi \to e^{i\theta}\psi$$



$$\psi \to e^{i2\theta}\psi$$

$$\psi \to e^{i2\theta}e^{i4\theta}\psi$$

$$e^{i2\theta}e^{i4\theta} = e^{i4\theta}e^{i2\theta}$$



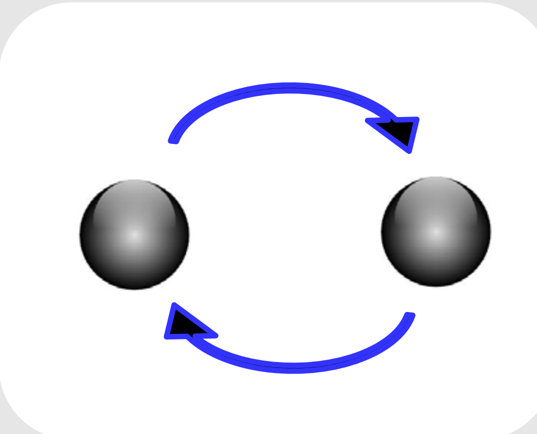
anyonic statistics in 2d → *non-abelian*

degenerate ground state

$$|\psi\rangle = \sum_i a_i |\psi_i\rangle = \vec{a}\cdot\vec{\psi}$$

$$|\psi\rangle = \vec{a}\cdot\vec{\psi} \rightarrow (U\vec{a})\cdot\vec{\psi}$$

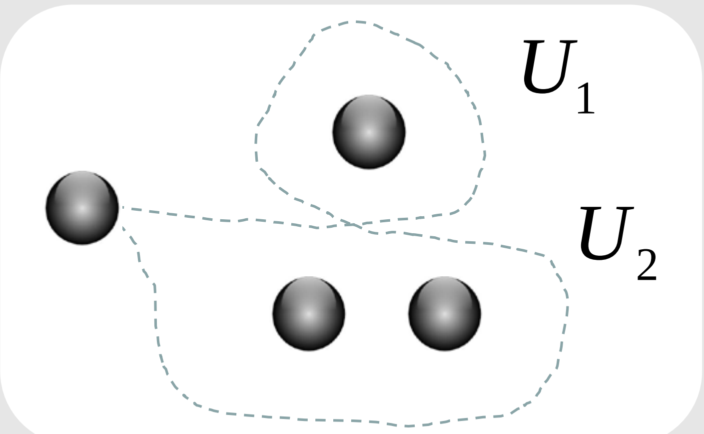
exchange → unitary



non-abelian anyons

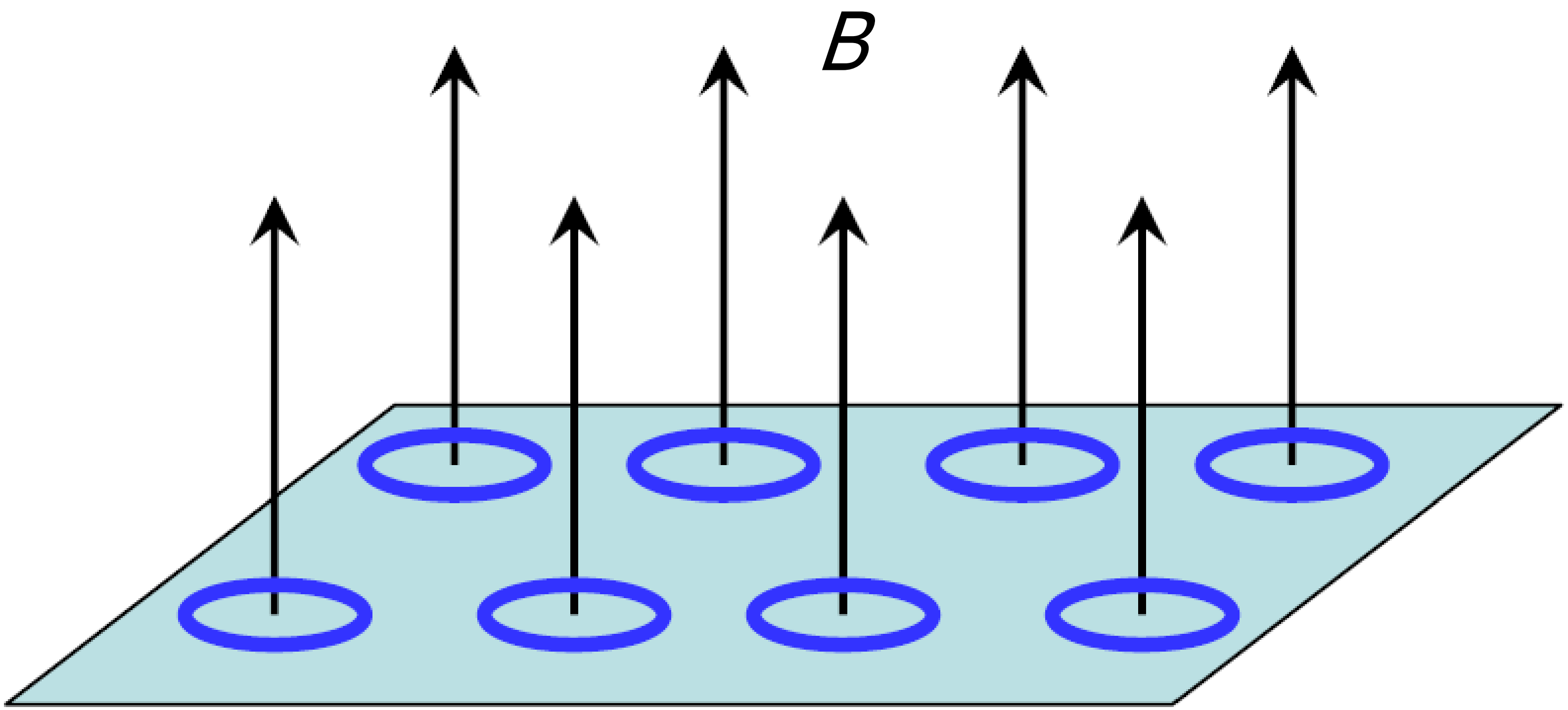
$$\psi \rightarrow U_1 U_2 \psi$$

$$U_1 U_2 \neq U_2 U_1$$



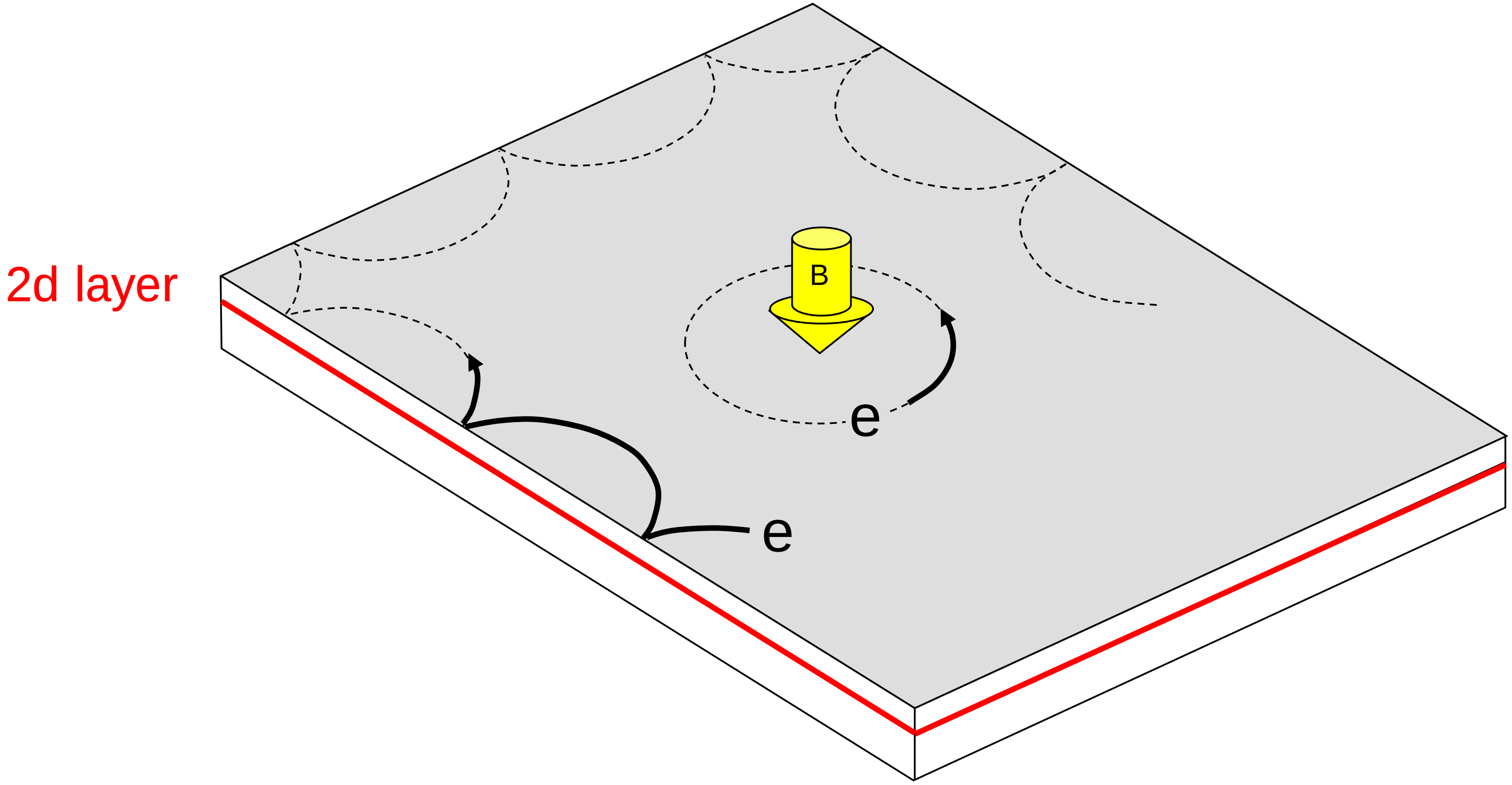
anyons in QHE

2DEG + magnetic field ... classical bulk



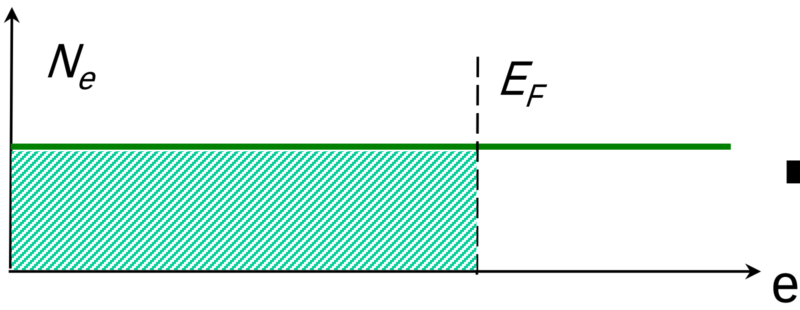
most convenient picture...

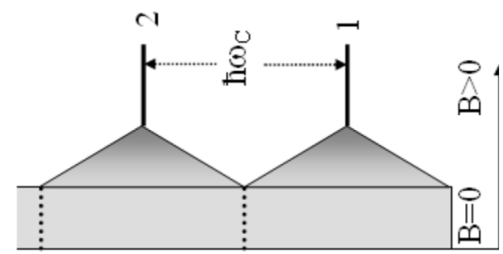
2DEG + magnetic field ... classical edge

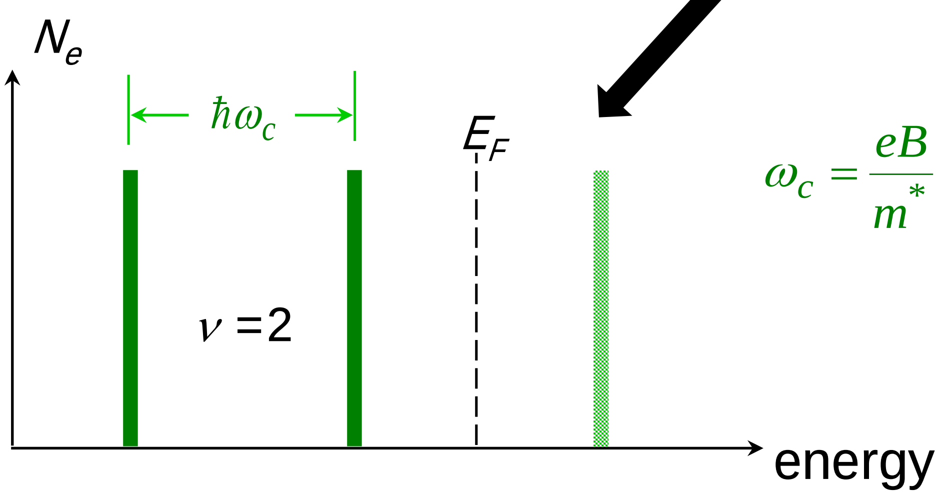


quantizing the Hall effect

no disorder







$$E_n = \hbar\omega_c(n + \tfrac{1}{2})$$

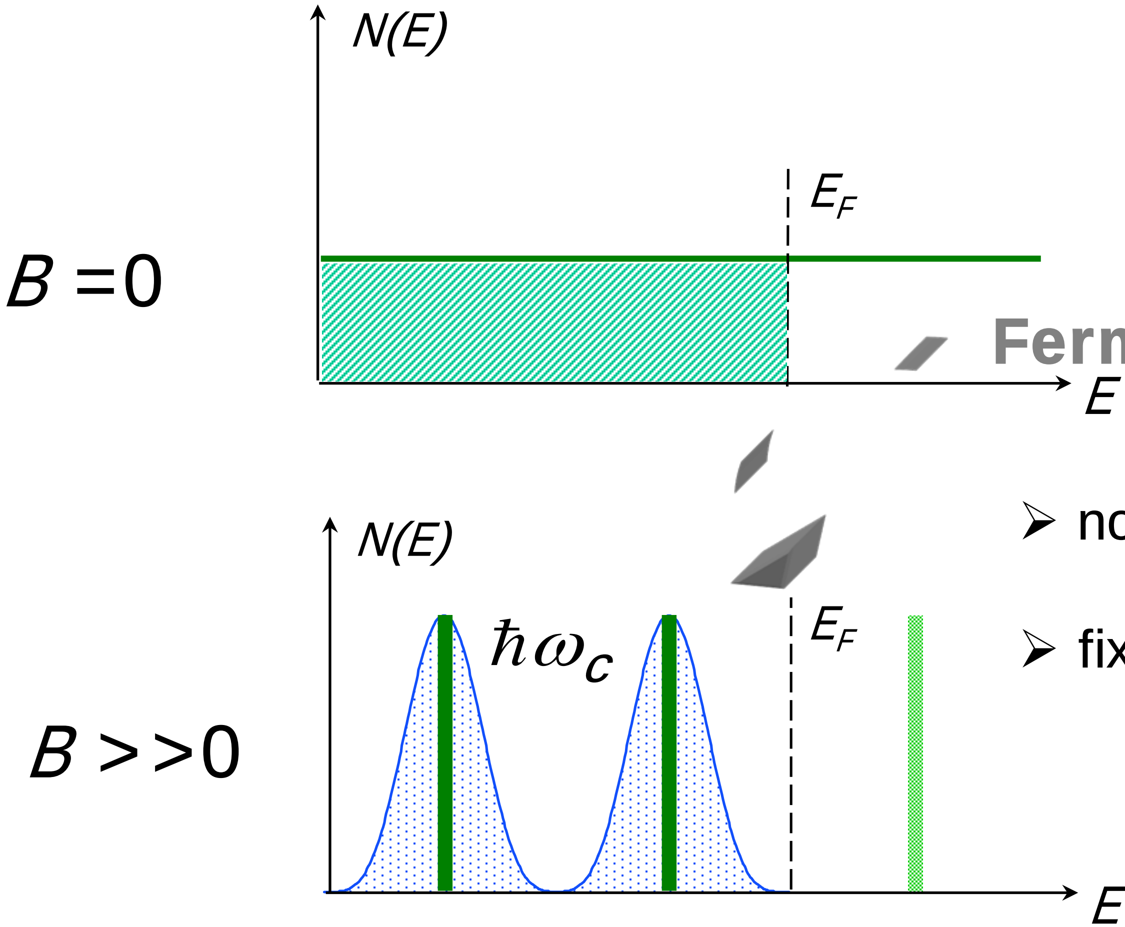
$$\omega_c = \frac{eB}{m^*}$$

$$l_B^2 = \frac{\hbar}{eB}$$

$\Phi_0 = h/e$

ν = number of filled LL = number of electrons per *flux quantum*

with disorder



Fermi level in gap

- no dissipation.........$R_{xx} = 0$
- fixed free carriers....$R_H = const.$

$\nu = 2$; number of filled LLs

choice of gauges for $\vec{A}$

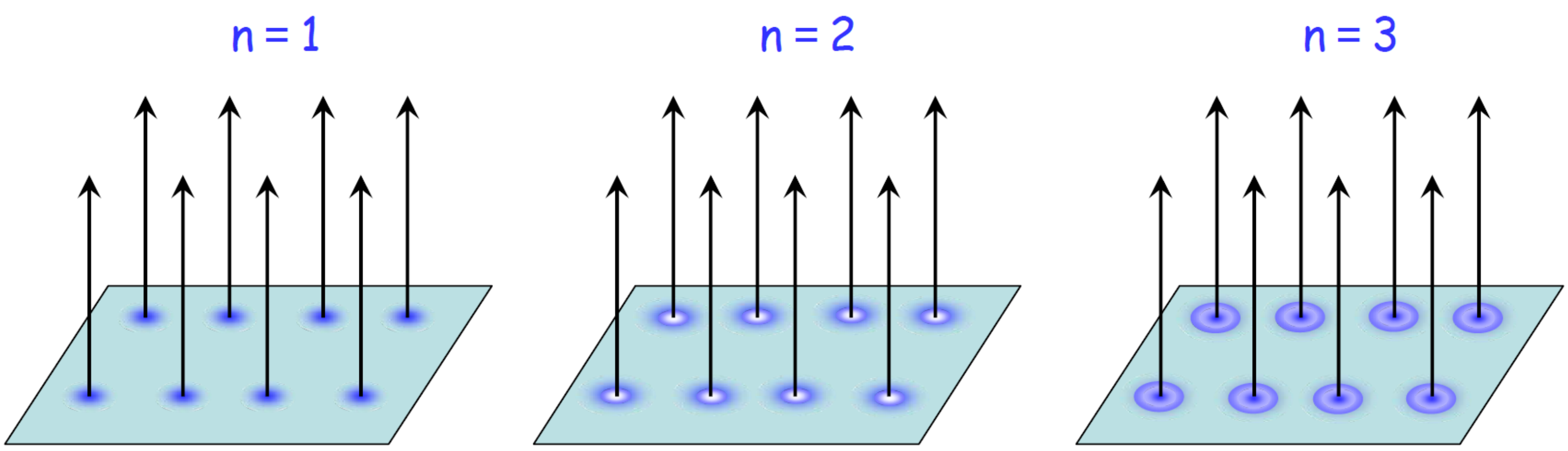
$$H = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2$$

symmetric (circular) gauge $\vec{A} = (A_x, A_y) = \frac{B}{2}(-y, x)$

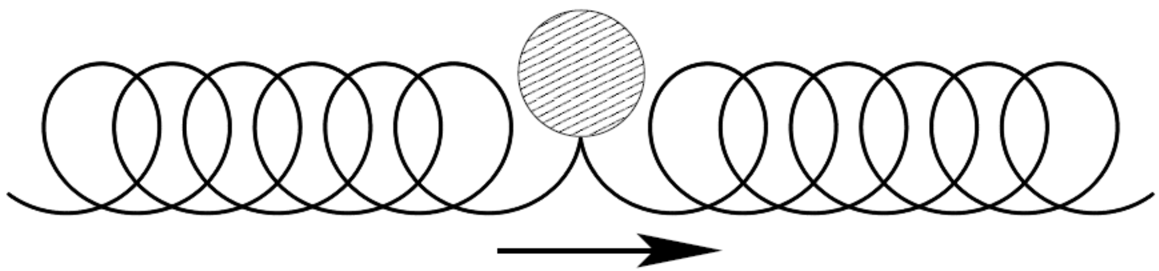
2DEG + magnetic field ... quantum edge

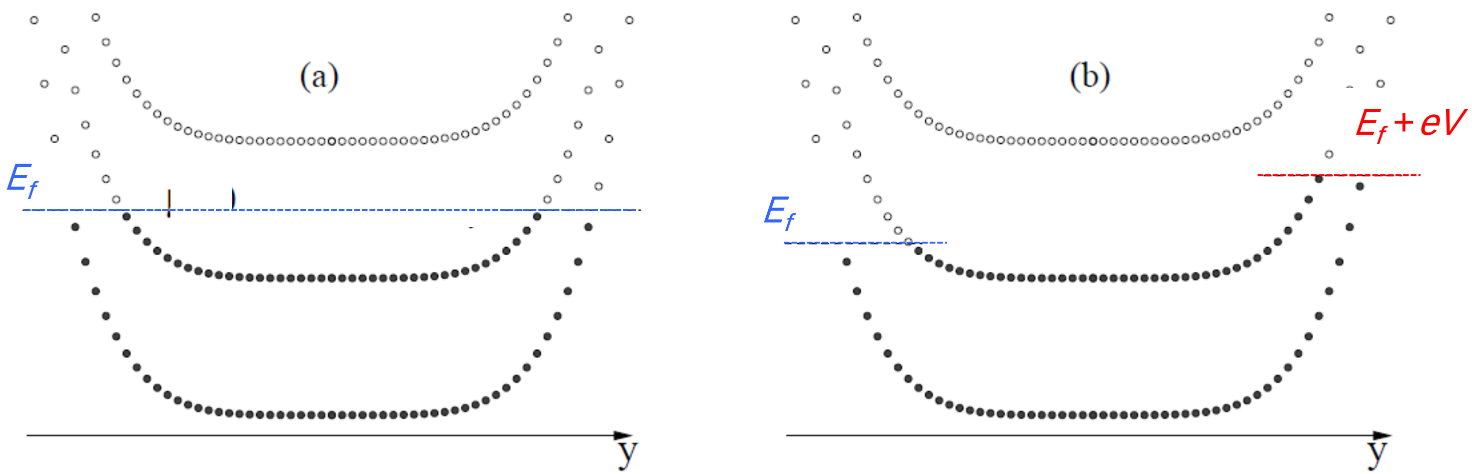
convenient gauge (for interference)

Landau levels...resembling classical orbits



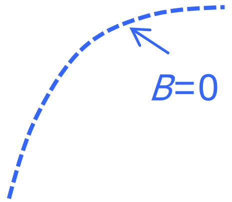
edge modes   immune to back scattering



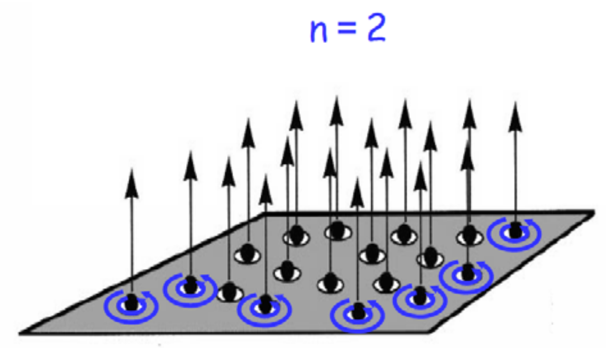


1d edge channel carries $$I = \frac{e^2}{h} V$$

2DEG + magnetic field … quantum edge

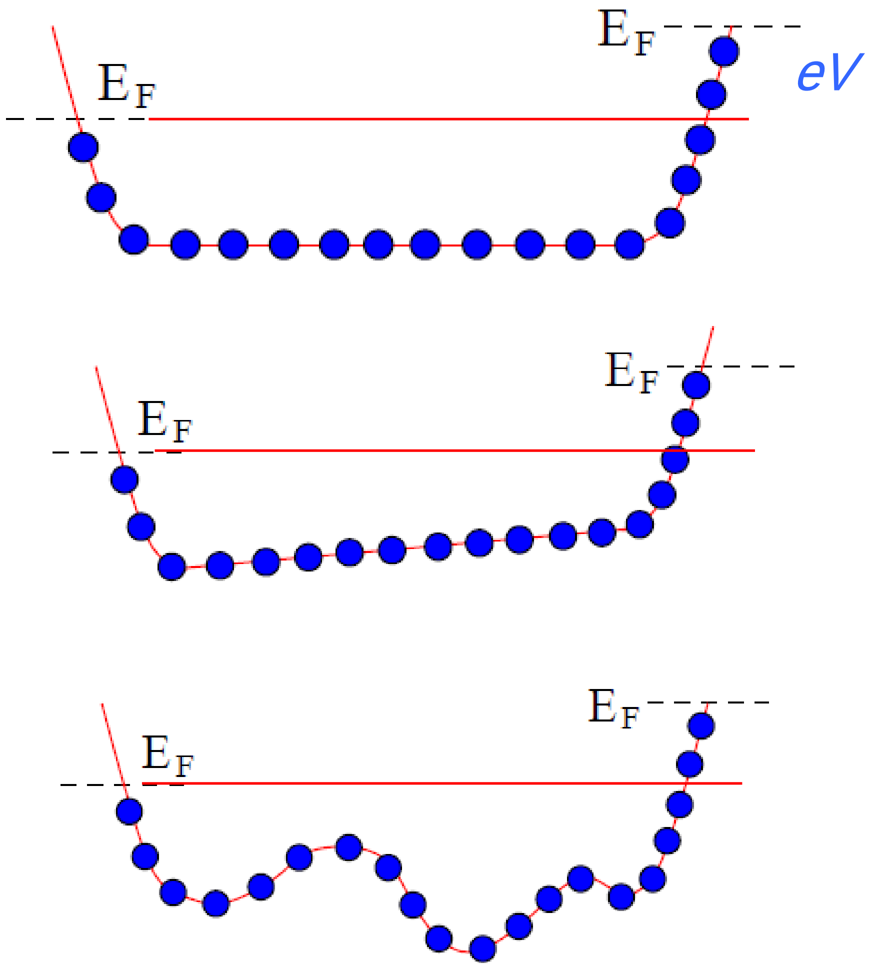


ν = number of electrons per *flux quantum*



Integer Quantum Hall Effect :
n electrons circle around **one** flux quantum
(more electrons than flux quanta).

edge current & bulk current



bulk current = non dissipative

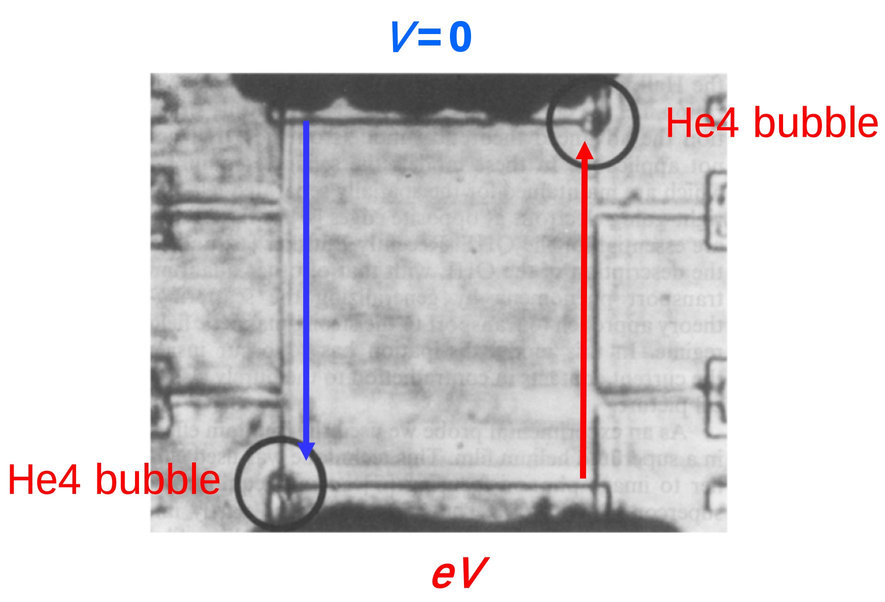
edge current = $e^2 V/h$

ballistic, but with energy dissipation hot spots

Imaging of the dissipation in quantum-Hall-effect experiments

U. Klaß, W. Dietsche, K. von Klitzing, and K. Ploog
Max-Planck-Institut für Festkörperforschung, W-7000 Stuttgart 80, Federal Republic of Germany

Received October 18, 1990 Z. Phys. B – Condensed Matter 82, 351–354 (1991)



will return to hot spots later...

in the beginning ... Si MOSFET

VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1980

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing
Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

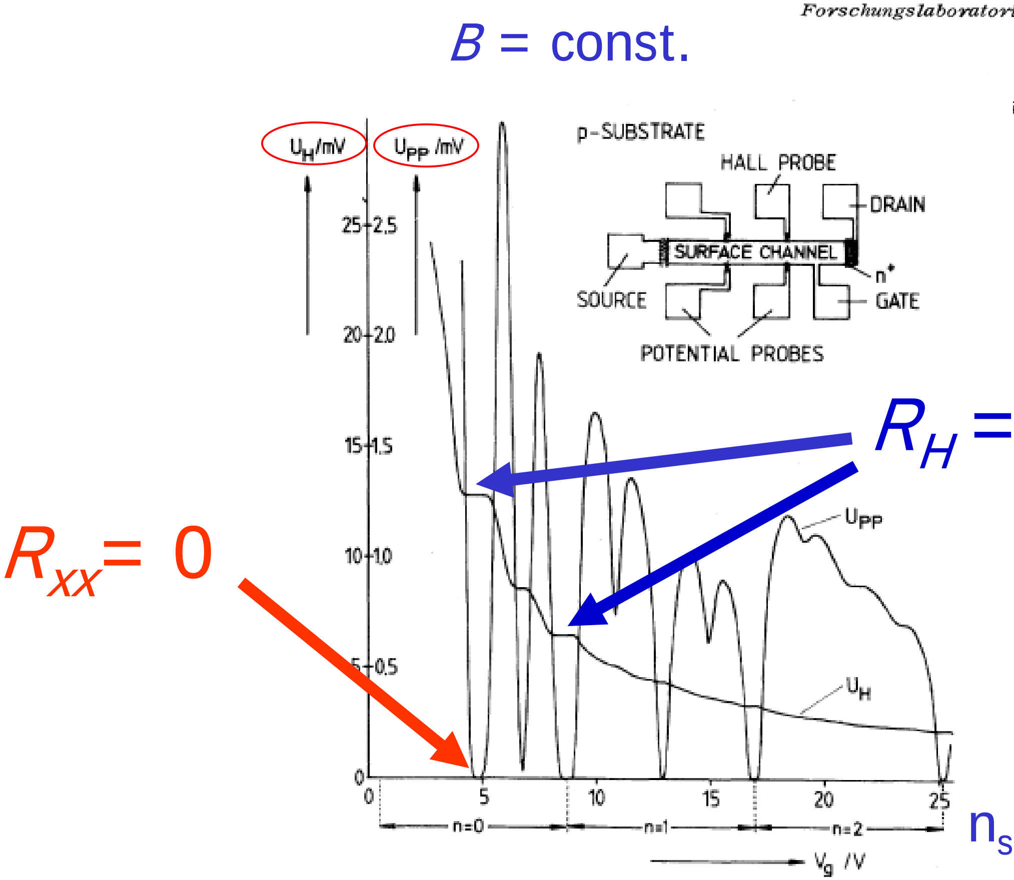
and

G. Dorda
Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany

and

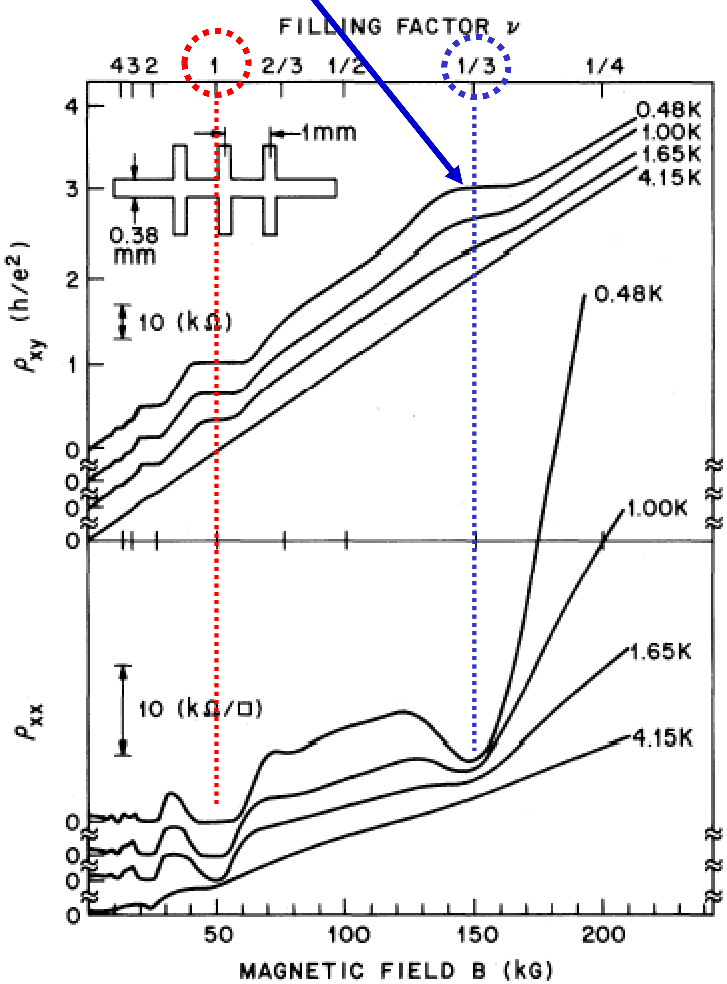
M. Pepper
lish Laboratory, Cambridge CB3 0HE, United Kingdom

(Received 30 May 1980)



continued with… GaAs MODFET

$R_H = (e^2/3h)^{-1}$



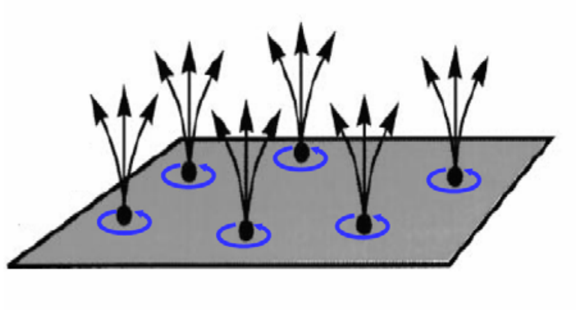
VOLUME 48, NUMBER 22 PHYSICAL REVIEW LETTERS 31 MAY 1982

Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,[a],[b] H. L. Stormer,[a] and A. C. Gossard
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 5 March 1982)

tion. Our observation of a quantized Hall resistance of $3h/e^2$ at $\nu = \frac{1}{3}$ is a case where Laughlin's argument breaks down. If we attribute it to the presence of a gap at E_F when $\frac{1}{3}$ of the lowest Landau level is occupied, his argument will lead to quasiparticles with fractional electronic charge of $\frac{1}{3}$, as has been suggested for $\frac{1}{3}$-filled quasi one-dimensional systems.[21]

ν = number of electrons per *flux quantum*

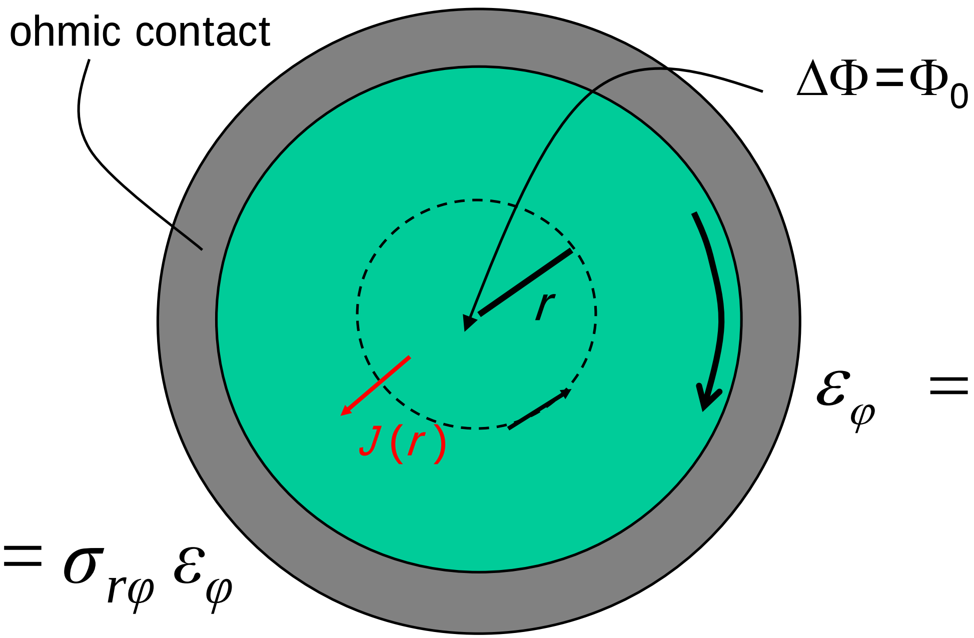


n = 1/3

Fractional Quantum Hall Effect n = 1/**m** :
One electron circles around **m** flux quanta
(more flux quanta than electrons).

Each flux quantum gets a fraction of the electron.

adiabatic ν =1/3...



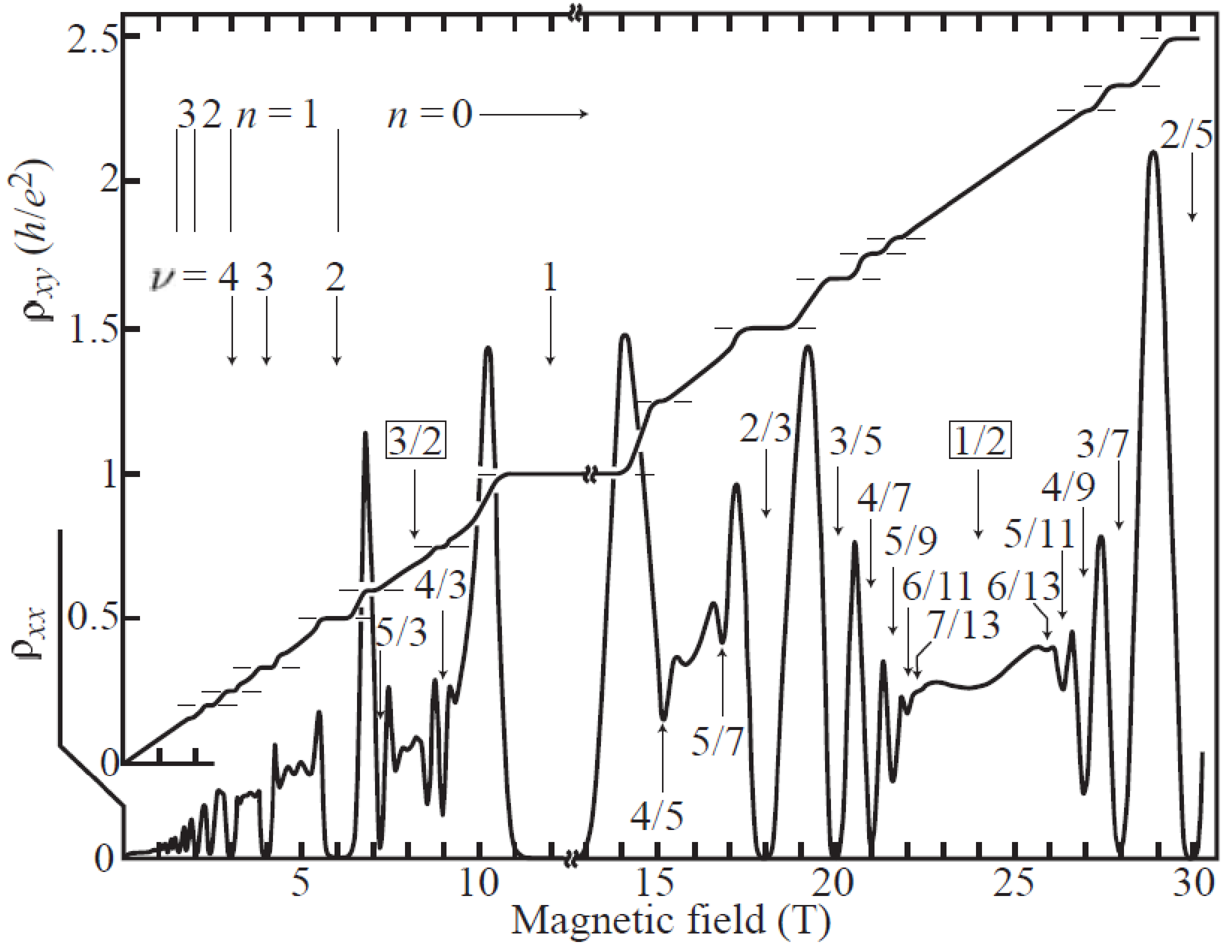
$$\varepsilon_{\varphi} = \frac{1}{2\pi r}\frac{\Phi_o}{\Delta t}$$

$$J(r) = \sigma_{r\varphi}\,\varepsilon_{\varphi}$$

$$\sigma_{r\theta} = \frac{1}{3}\frac{e^2}{h}$$

$$q = I(r)\Delta t = 2\pi\, r J(r)\Delta t = e/3$$

with time...

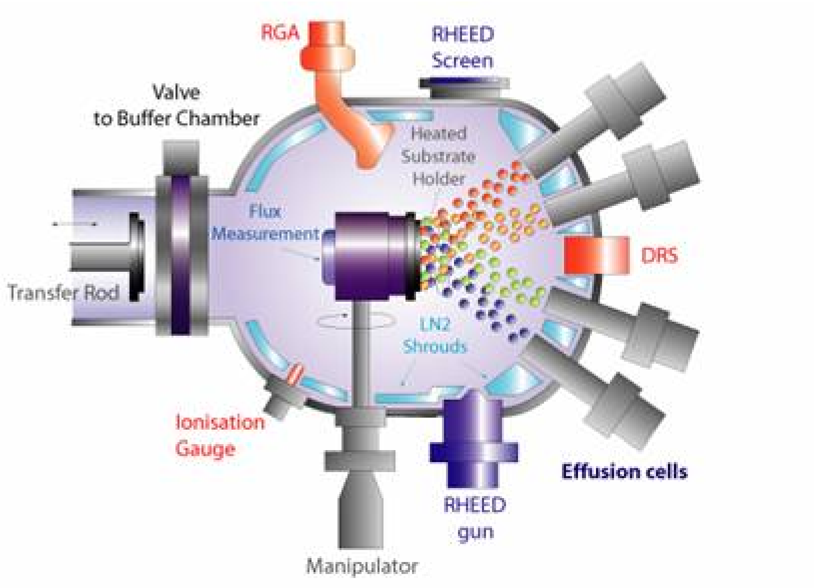


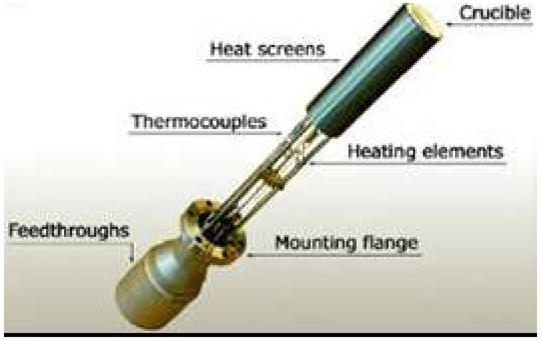
2.5
2
1.5
1
0.5
0
0
ρ_{xy} (h/e^2)
ρ_{xx}
3 2 $n = 1$
$n = 0$
$\nu = 4$ 3
2
1
3/2
4/3
5/3
4/5
5/7
2/3
3/5
4/7
5/9
6/11
6/13
7/13
1/2
5/11
4/9
3/7
2/5
5
10
15
20
25
30
Magnetic field (T)

hi quality 2DEG

growing GaAs – AlGaAs heterostructures

MBE growth



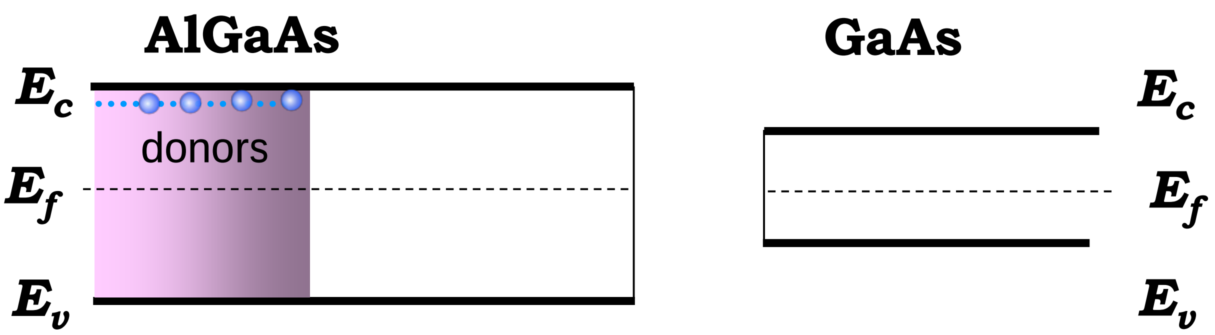


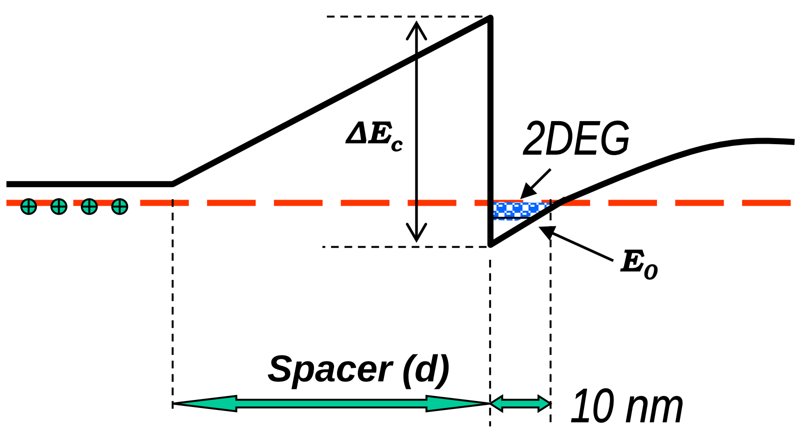
Molecular Beam Epitaxy (MBE)

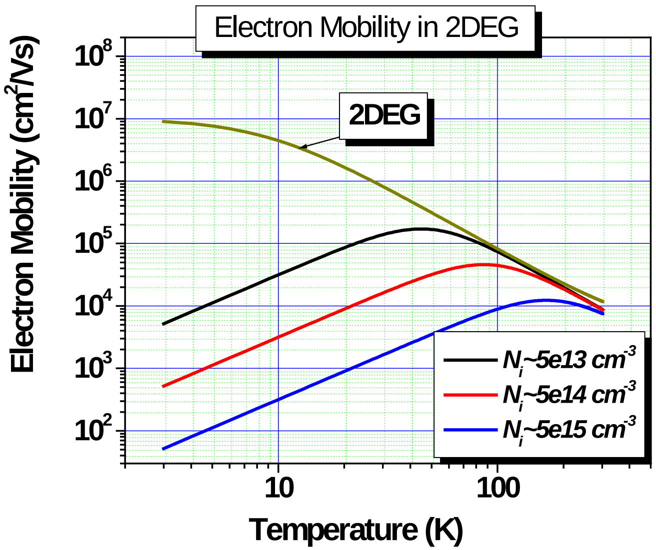
pressure ~$1 \div 10 \times 10^{-12}$ torr

growth rate ~1 micron/Hour → 1ML/sec

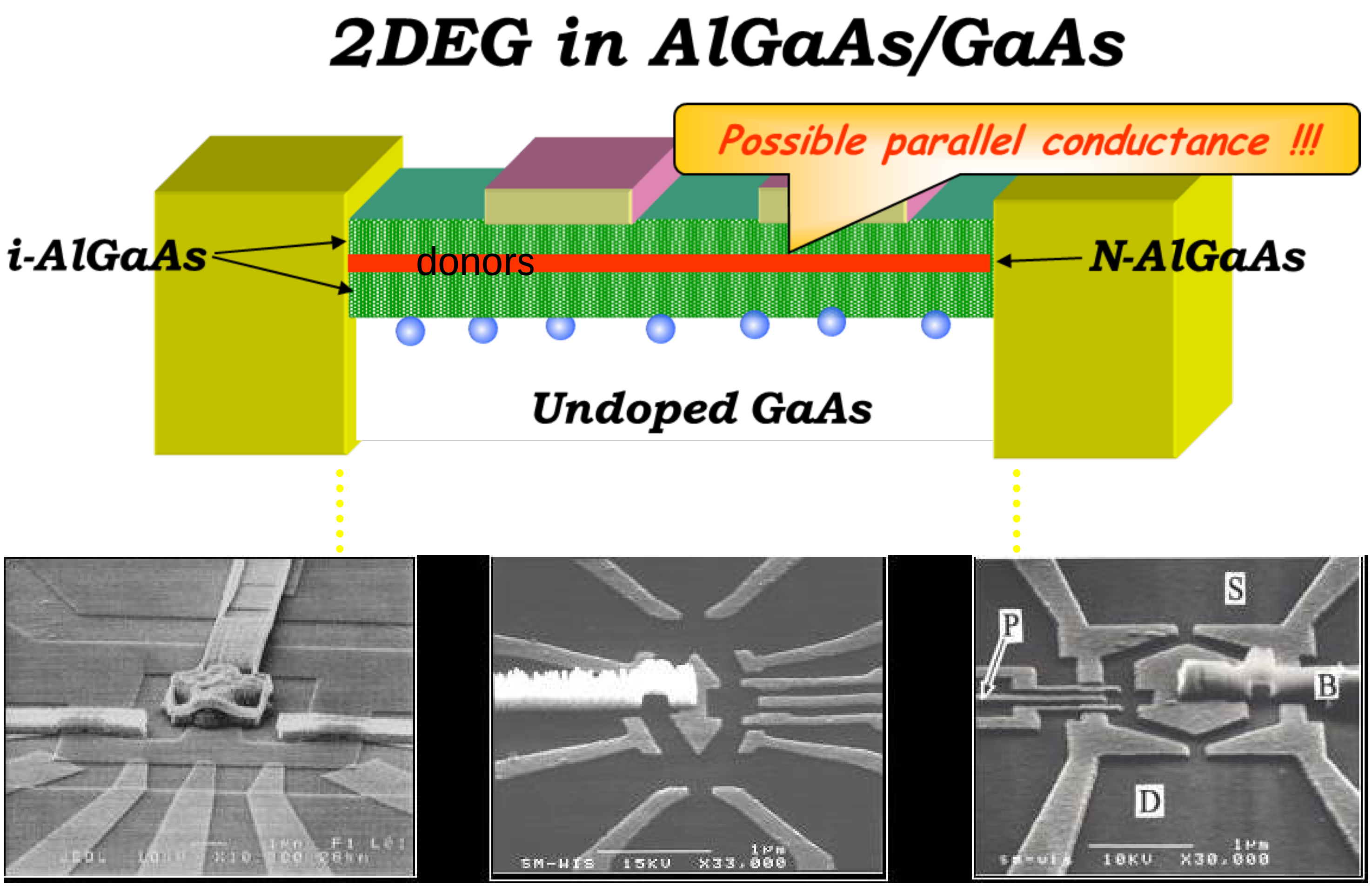
making pure 2DEG



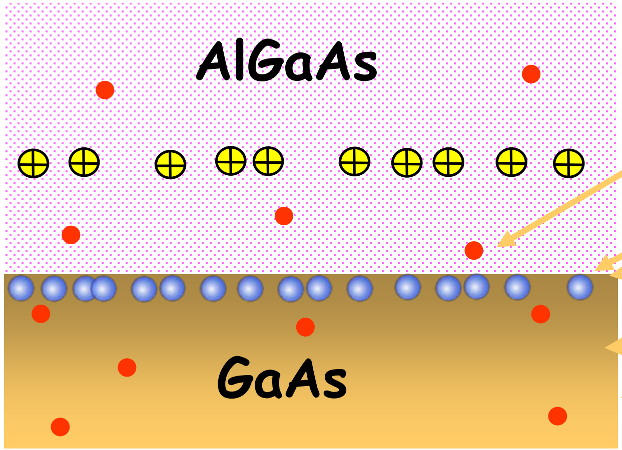




typical structure



mobility: scattering mechanisms



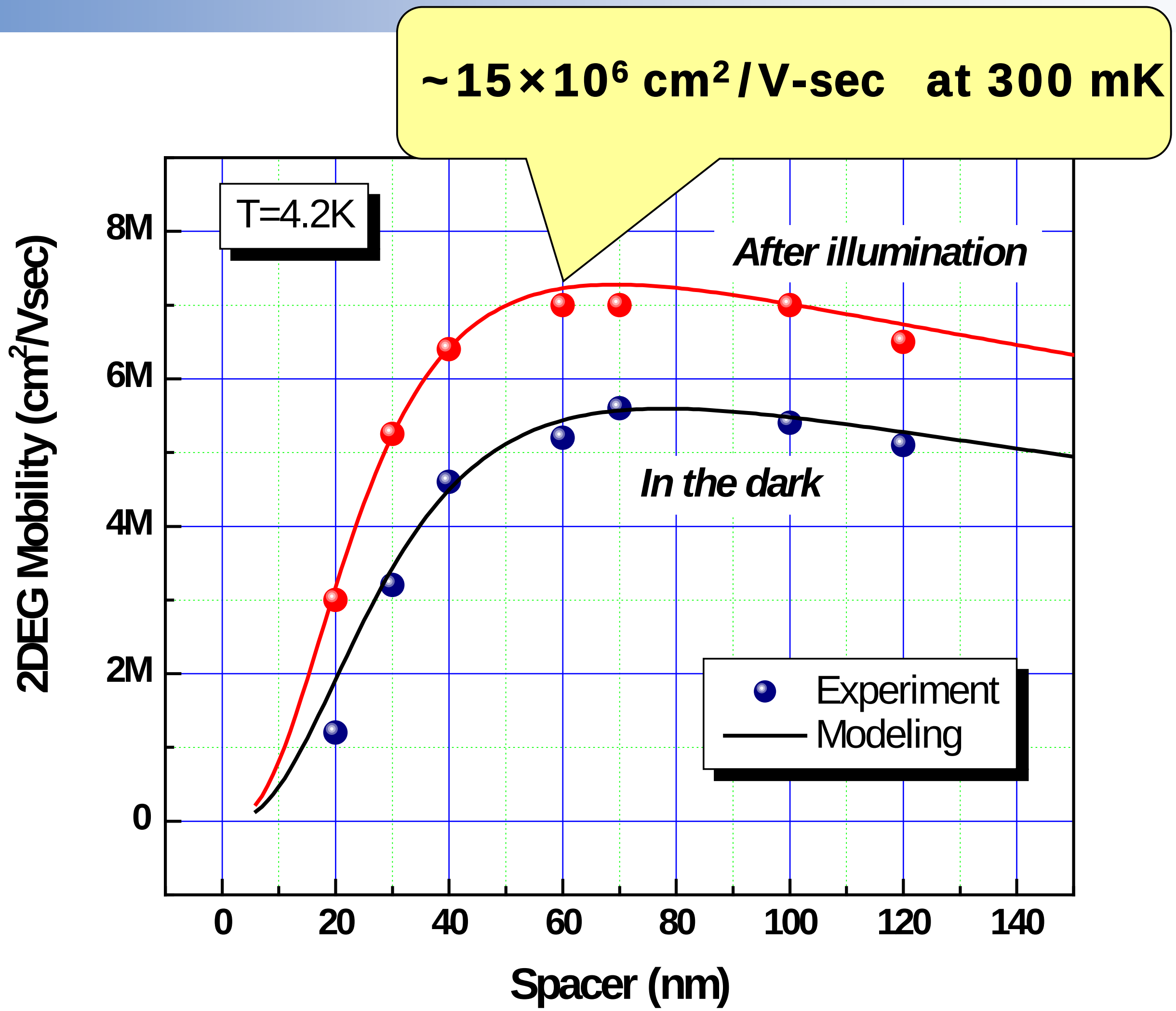
1. *remote Ionized Impurities*
2. *interface Roughness*
3. *alloy scattering*
4. *background Impurities*
5. *phonon Scattering*

$Si : N^+$ doping in AlGaAs

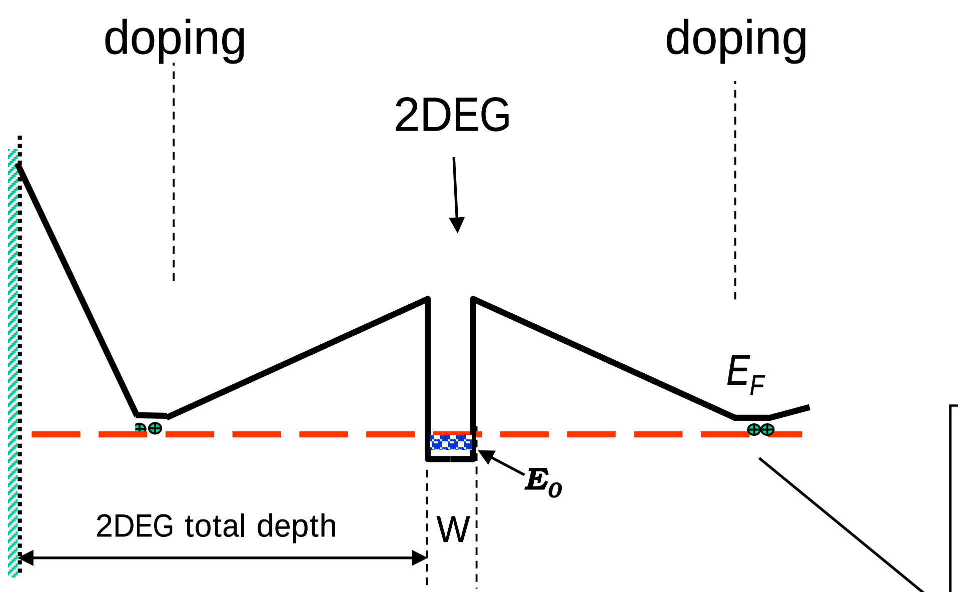
$$\frac{1}{\mu} = \sum_i \frac{1}{\mu_i}$$

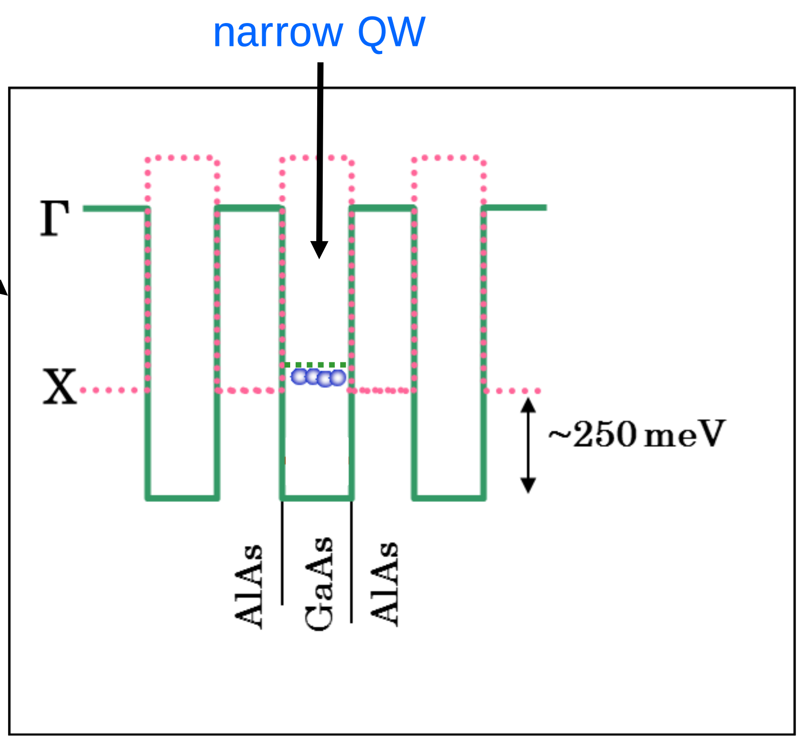
deep DX centers – electrons freeze at 100K

mobility…..dependence on spacer



superlattice type doping





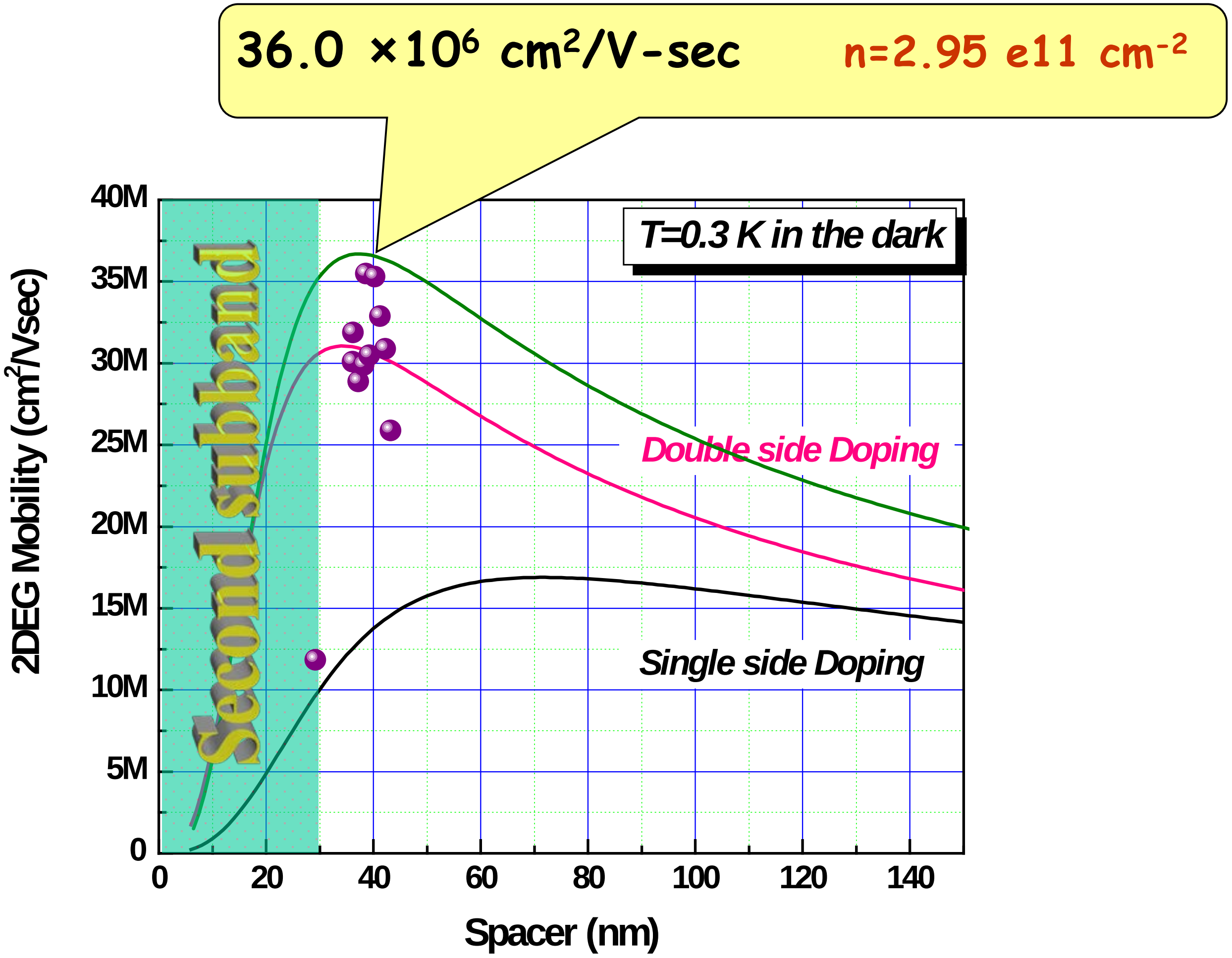
Si : N+ doping in narrow GaAs QW

electrons spill over to AlAs X-valley

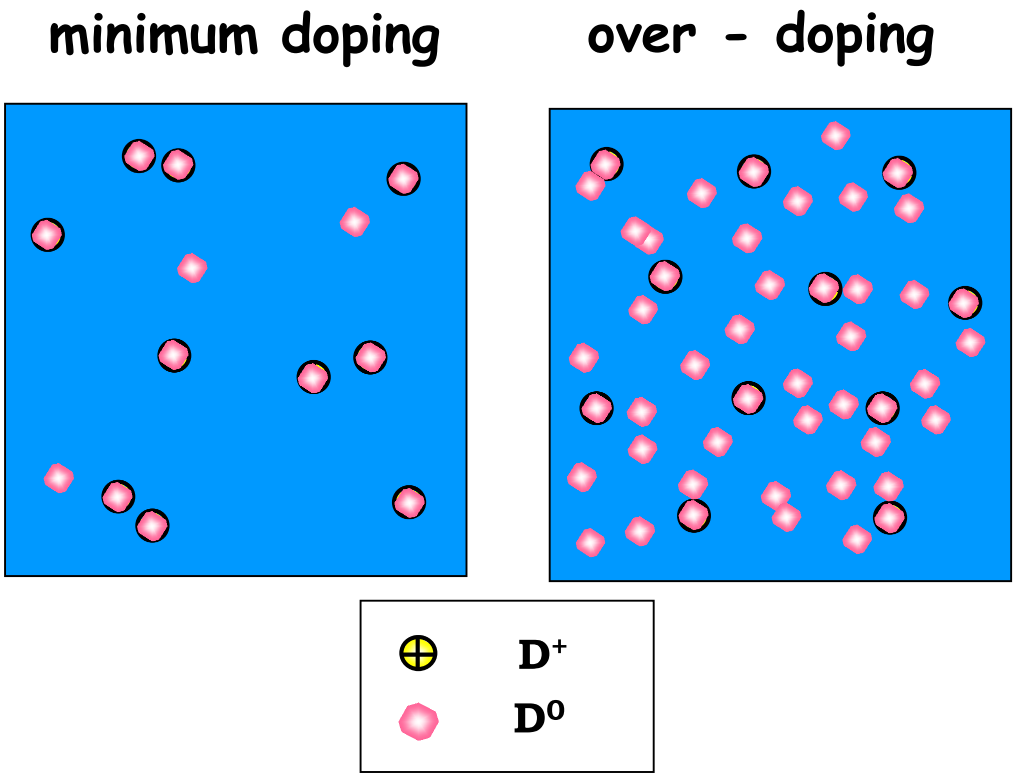
electrons mobile - but very low mobility

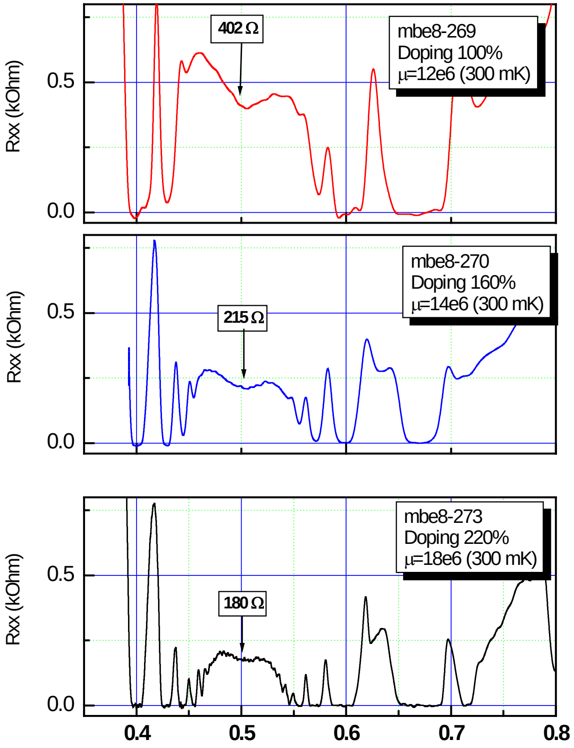
effective screening of donors

experimental data

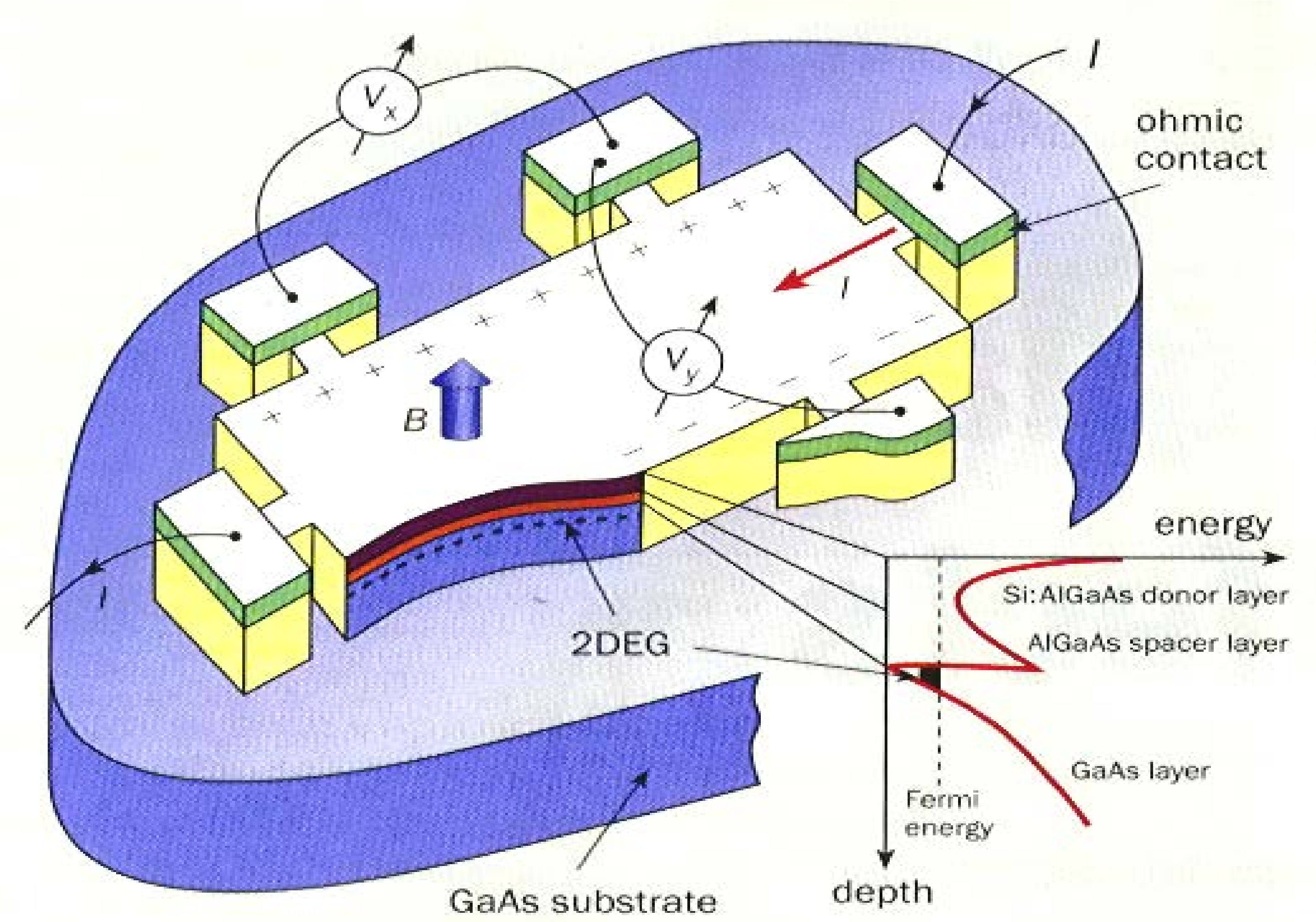


donor correlations

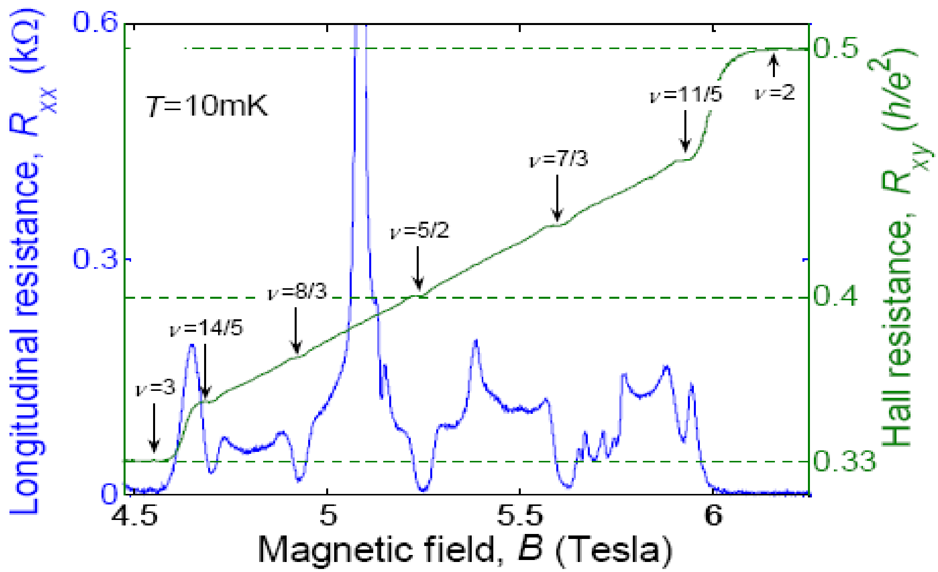




standard Hall-bar



typical 1st excited LL

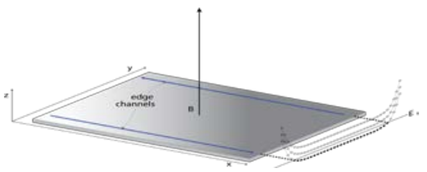


n_e= 3.2×10^{11} cm^{-2} μ=30.5×10^{6} cm^{2}/V-sec

in dark

types of edge modes

edge modes



edge of IQHE ....integer $\nu = \mathbf{1, 2, 3},...$

edge of FQHE ....fractional

abelian states .... particle-like $\nu = \mathbf{1/3, 2/5},..,$

hole-conjugate $\nu = \mathbf{2/3, 3/5, 4/7},...$

non-abelian state .... $\nu = \mathbf{5/2},...$

edge modes

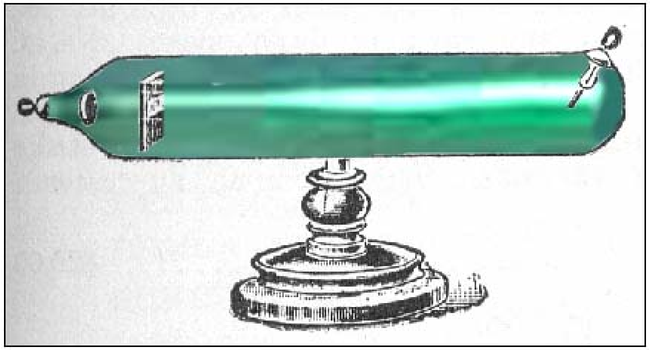
- *downstream* charge..............................particle-like states

  chirality of B field

- *downstream* + *upstream* (*neutral*) ......hole-conjugate & non-abelian states current fluctuations

shot noise

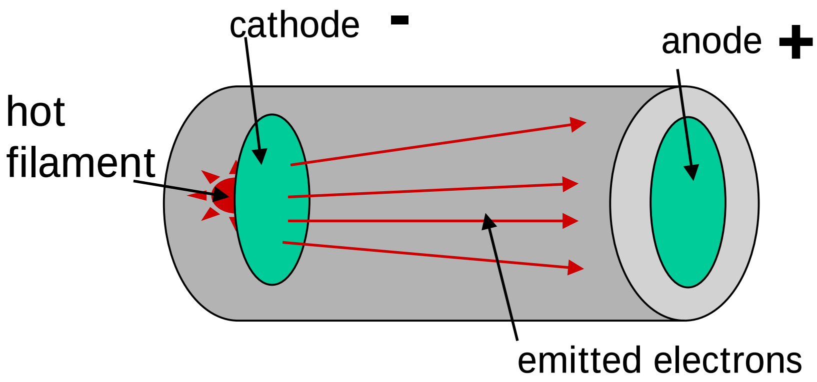
it started with - noise in vacuum tubes



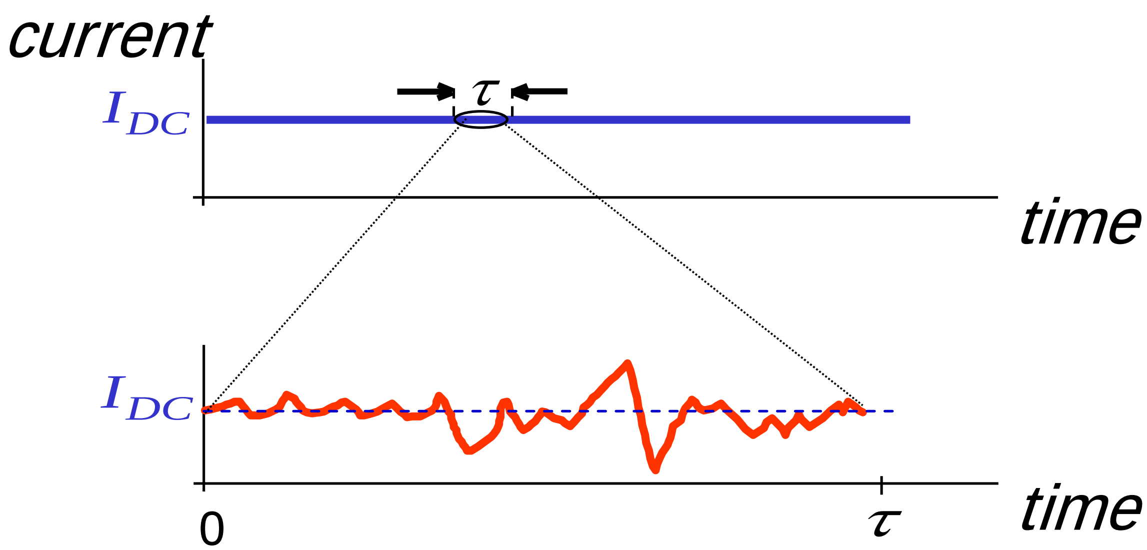
Schottky, 1918

noisy current in vacuum tubes

classical shot noise



classical shot noise

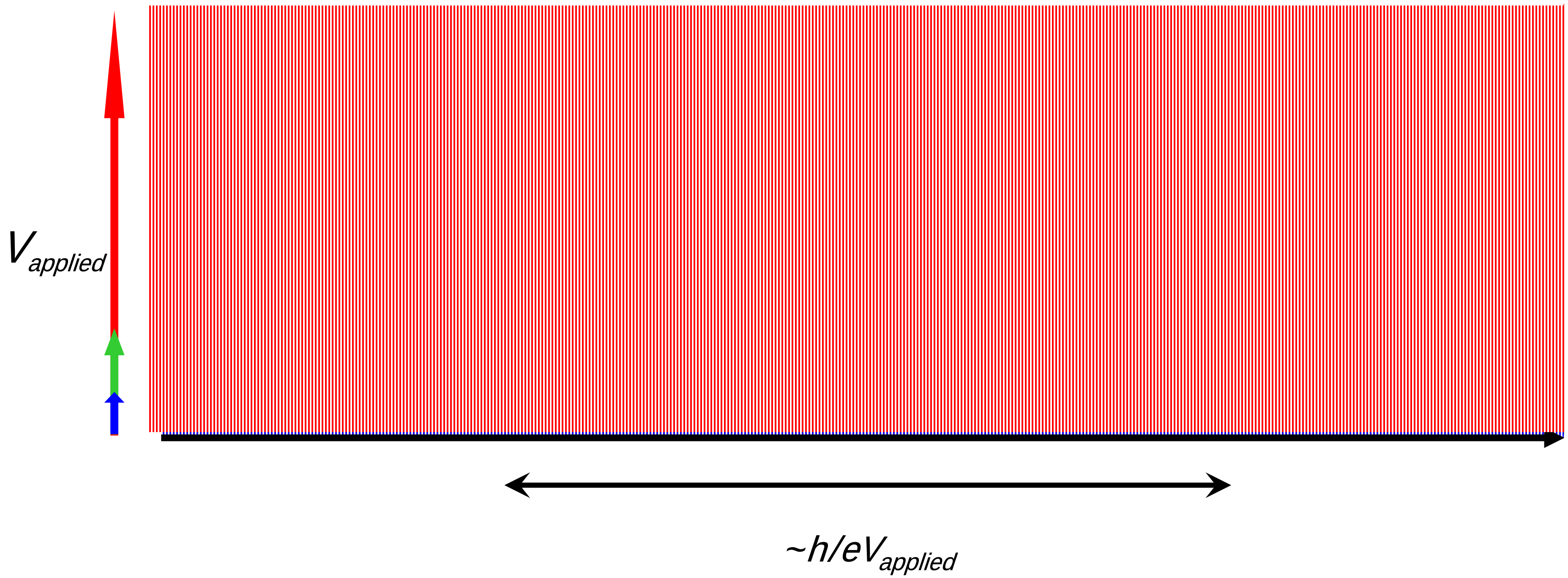


- large number of impinging electrons
- very small escape probability

$$S_i(0) = 2\,e\,I$$

spectral density (A^2/Hz)

shot noise = 0 …. full Fermi sea (un-partitioned electrons)



zero- temperature ordered electrons are noiseless ! Khlus 1987
Lesovik 1989

during measurement time τ ...... total charge transferred Q

$$Q = e\sum_{i=1}^{N} p_i \qquad p_i = 0,\ 1 \qquad < p_i > = t$$

charge fluctuations :

stochastic

$$< \Delta Q > = e\sum_{i=1}^{N} (p_i - < p_i >) \quad = \quad 0$$

$$< (\Delta Q)^2 > = e^2 < \left[\sum_{i=1}^{N} (p_i - < p_i >) \right]^2 >$$

$$= e^2 \sum_{i=1}^{N} < (p_i - < p_i >)^2 >$$

$$= e^2 N \cdot t(1-t) \qquad < p_i > = < p_i{}^2 > = t$$

(A²/Hz) $$S(0) = 2\frac{\langle (\Delta Q)^2 \rangle}{\tau} = 2eV\frac{e^2}{h}t(1-t) = 2eI_{transmitted}(1-t)$$

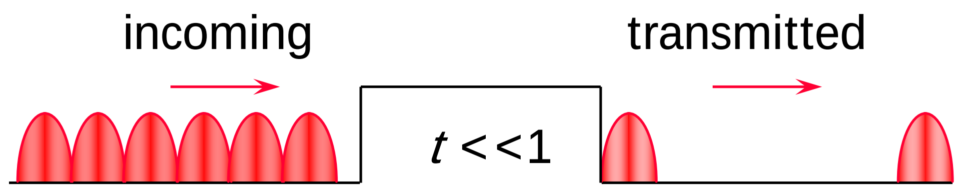
spectral density of current fluctuations

shot noise - single channel

poissonian

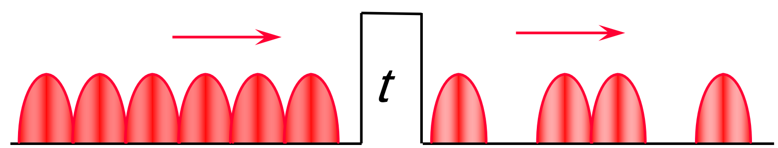
$S = 2eI$

Schottky formula

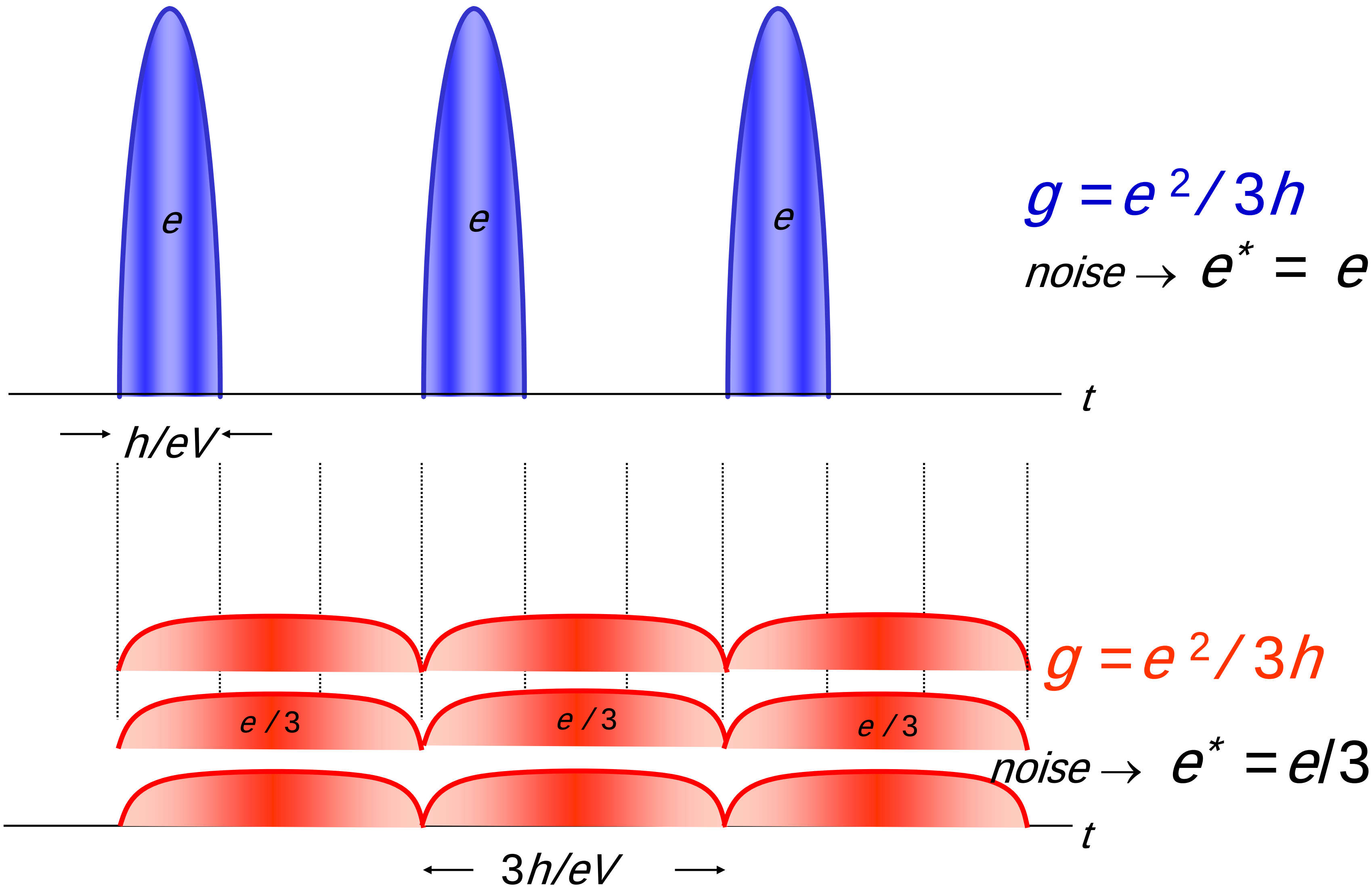


binomial

$S = 2eI(1 - t)$



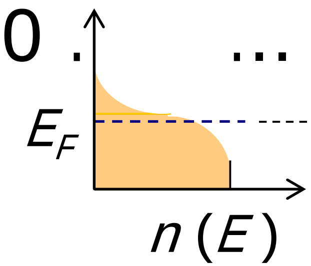
similar **conductance** - different **shot noise**

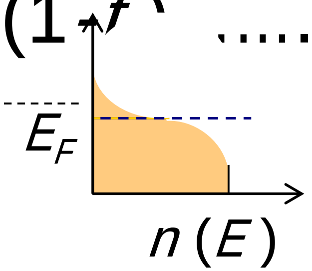


shot noise, $T > 0$

$V = 0$ .... $S_i(0) = 4k_BTg$ ....thermal

$T = 0$ ... $S_i(0) = 2qI(1-t)$ ......shot

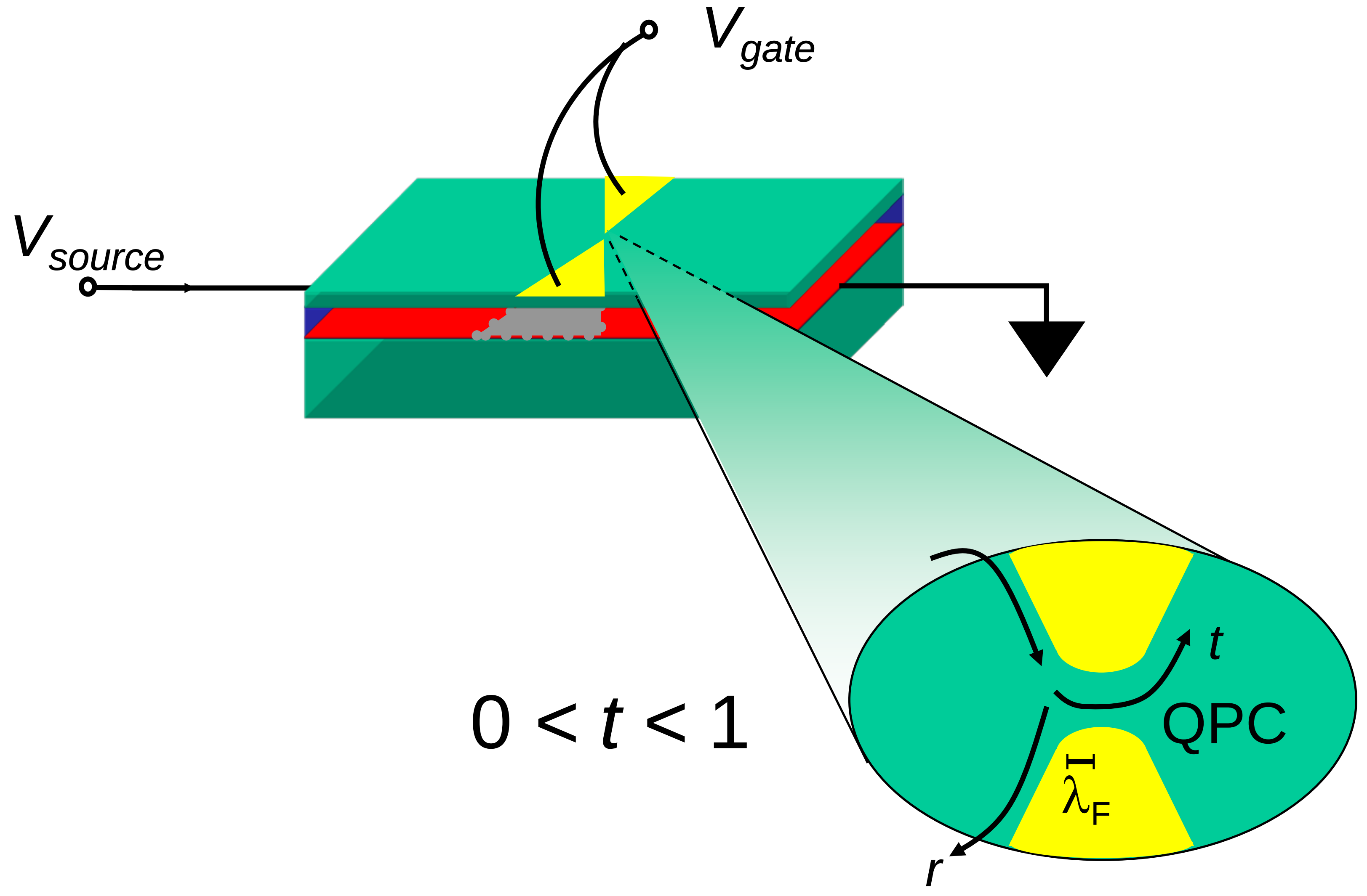




$V, T > 0$ ....total

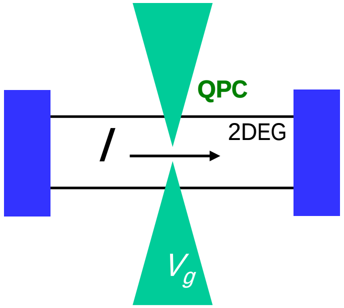
$$S_i(0) = 4k_BTg + 2qI(1-t)\left[\coth\left(\frac{qV}{2k_BT}\right) - \frac{2k_BT}{qV}\right]$$

quantum point contact (QPC)

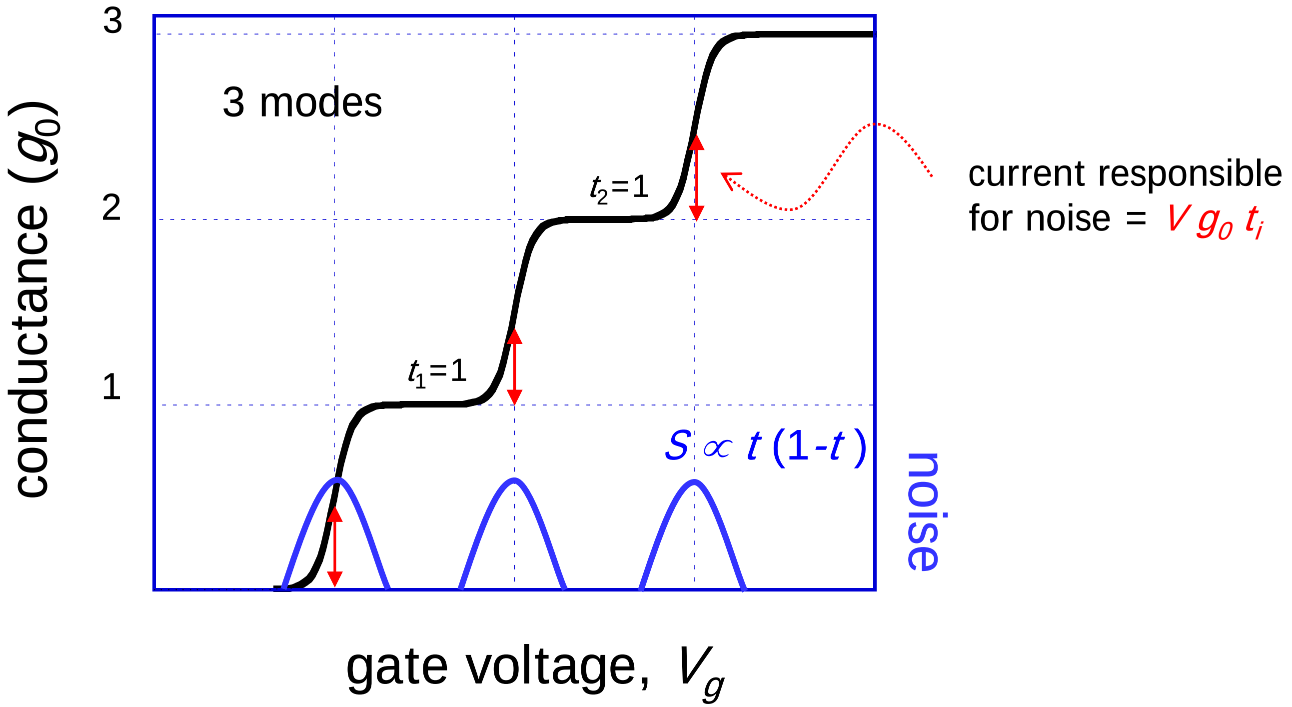


$0 < t < 1$

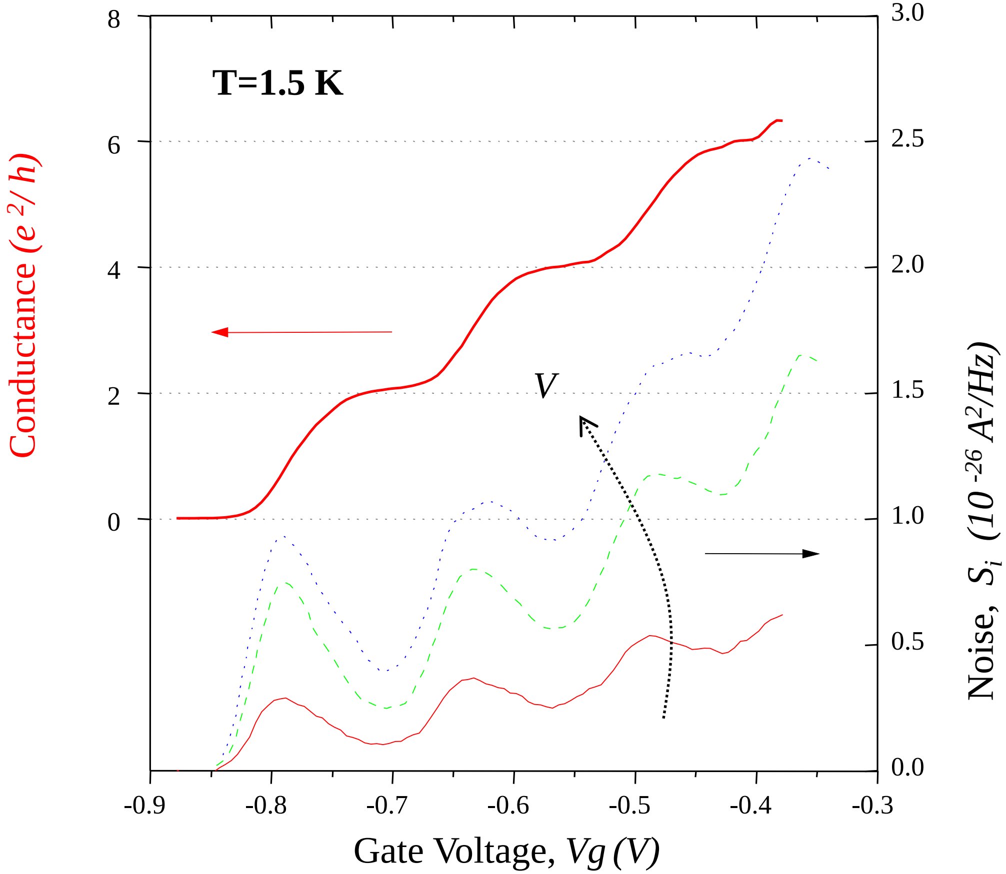
conductance and shot noise in QPC



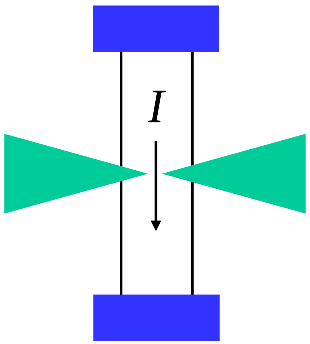
$$S_i(0)|_{T=0} = 2eV g_0 \sum_i t_i(1-t_i)$$



excess shot noise in QPC



two-terminal

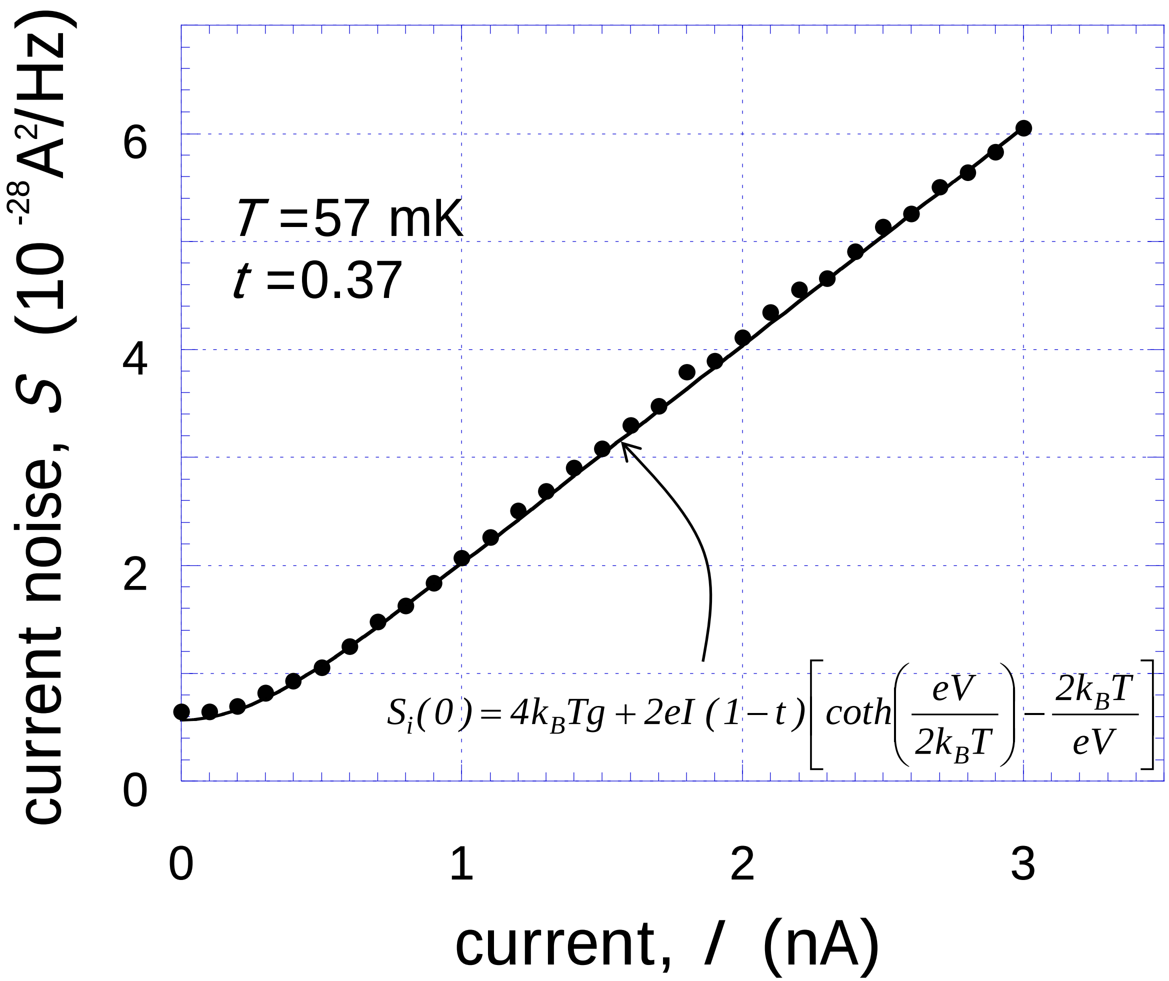


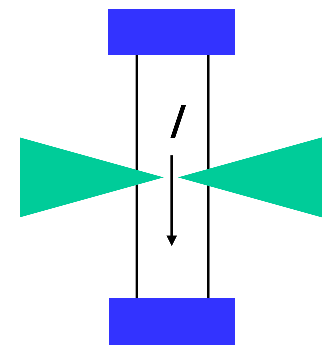
Reznikov et al. 1995

Kumar et al. 1996

shot noise in QPC

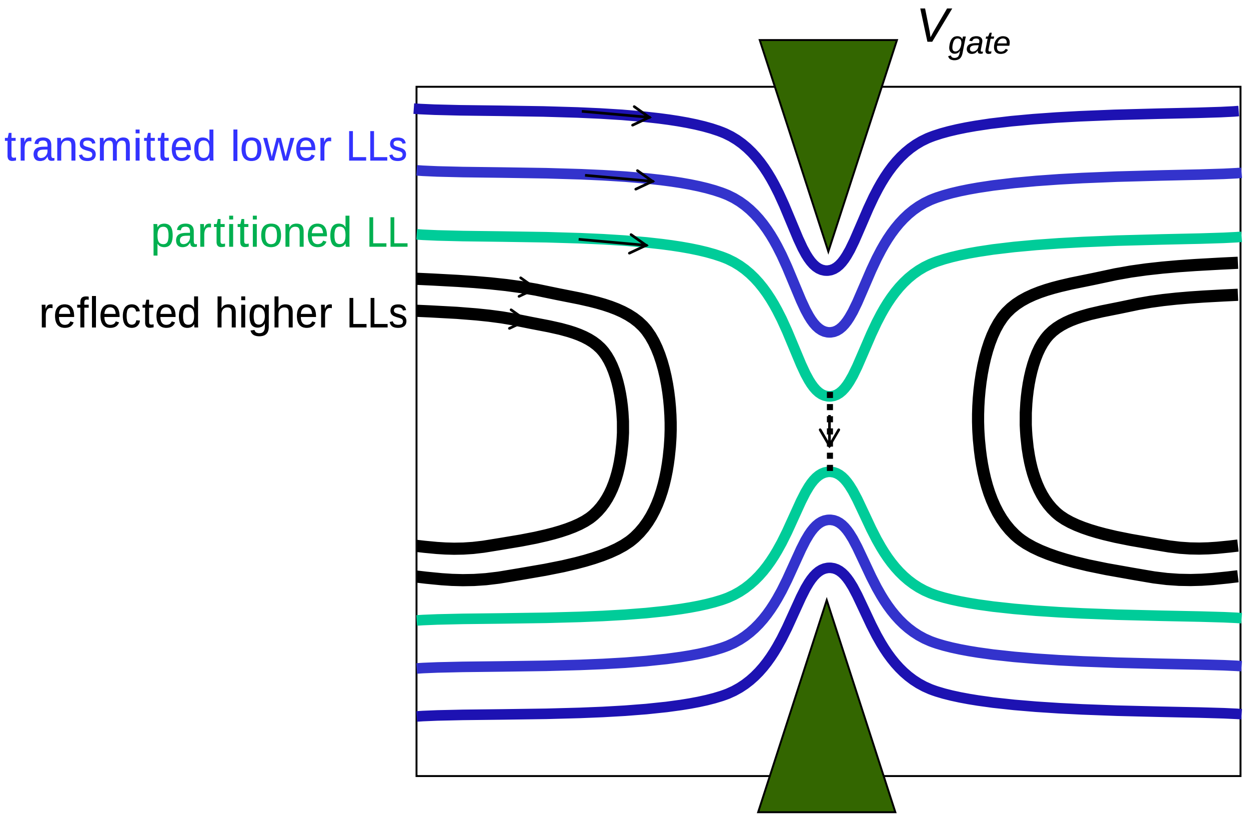
- experimental results -



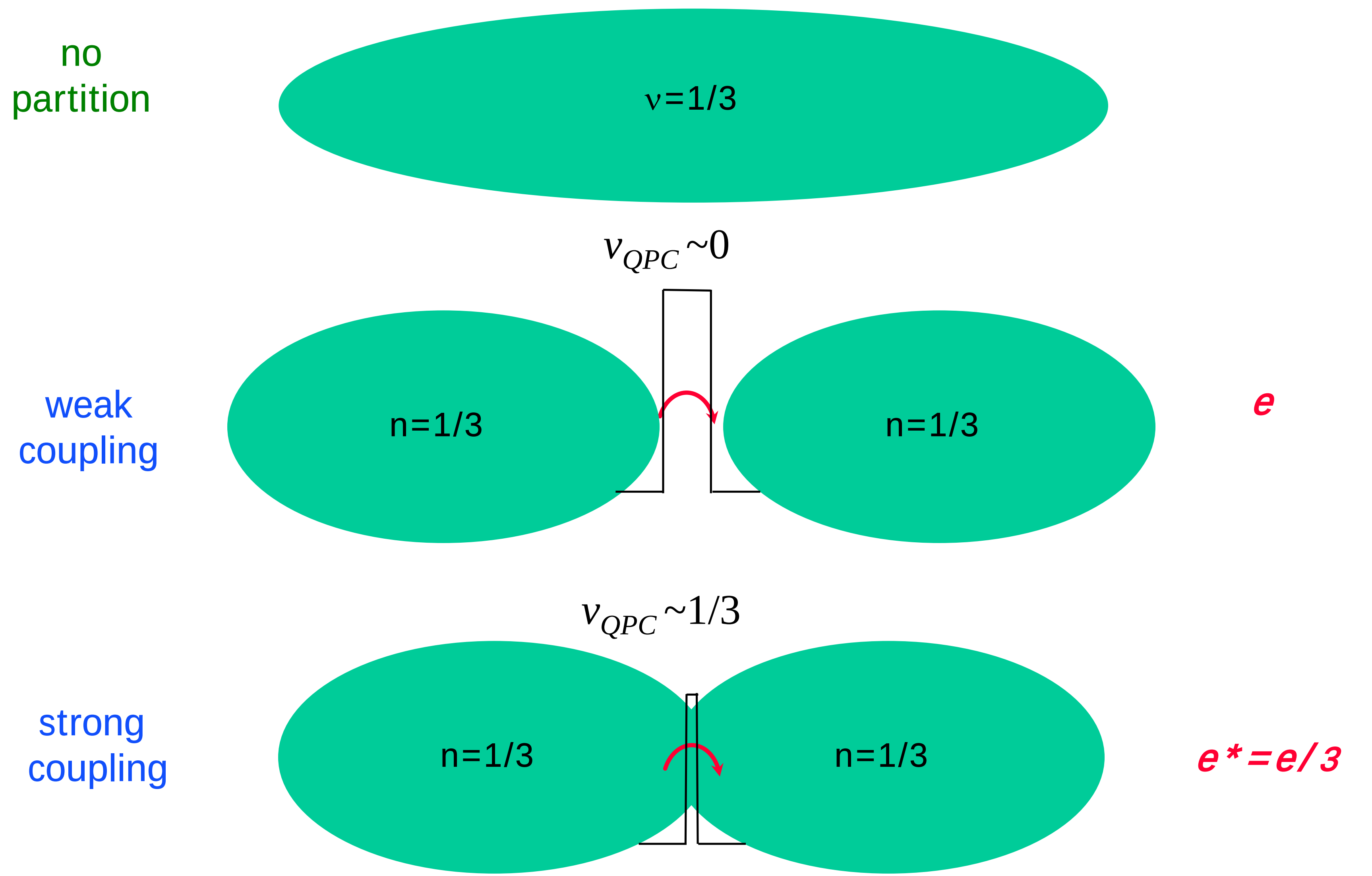


QHE

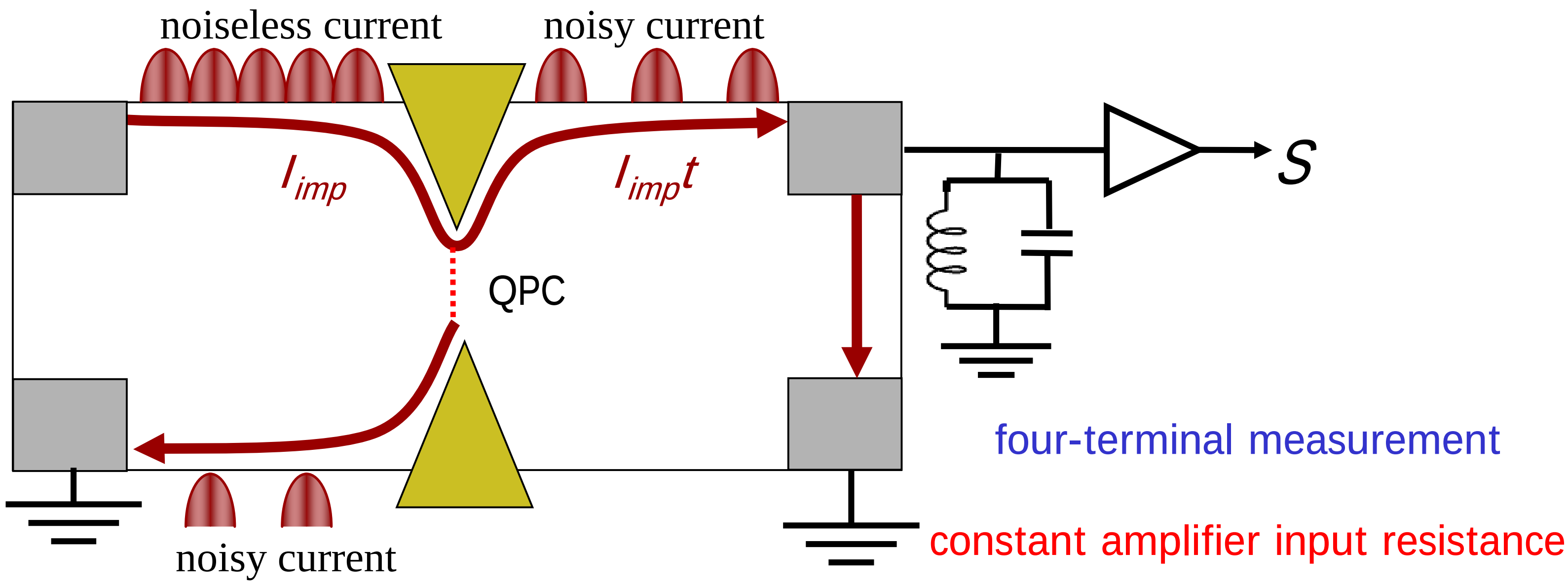
preferential backscattering of edge channels



will shot noise measure *e* or *e** ?



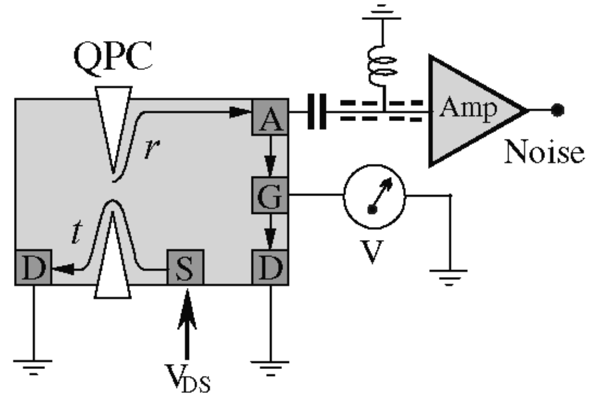
partitioning edge modes



FQHE

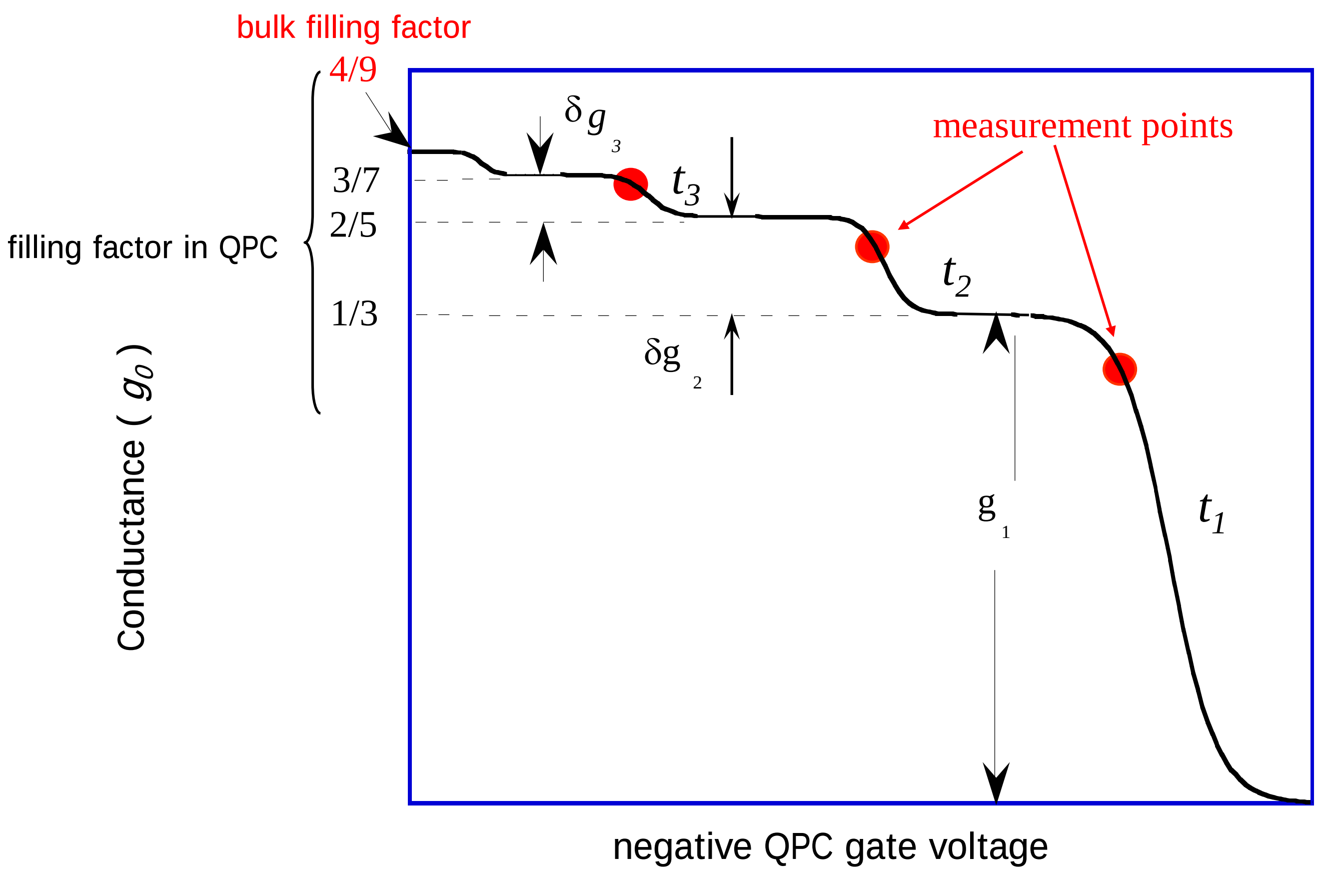
quasiparticles in edge modes are plasma-like waves - charge is not defined

shot noise measures the backscattered charge through the bulk (in the QPC)



amplifier input G_Q

non-linearity of QPC irrelevant



experimental considerations

2DEG : n_s=1.1x10^{11} cm^{-2} ; μ=4 x10^{6} cm^{2}/Vs

$S_i(0) = 4k_B T G_0$

shot noise signal $S_i(0)=2e^*I=10^{-29}$ A^2/Hz .... T^*~ 40 mK

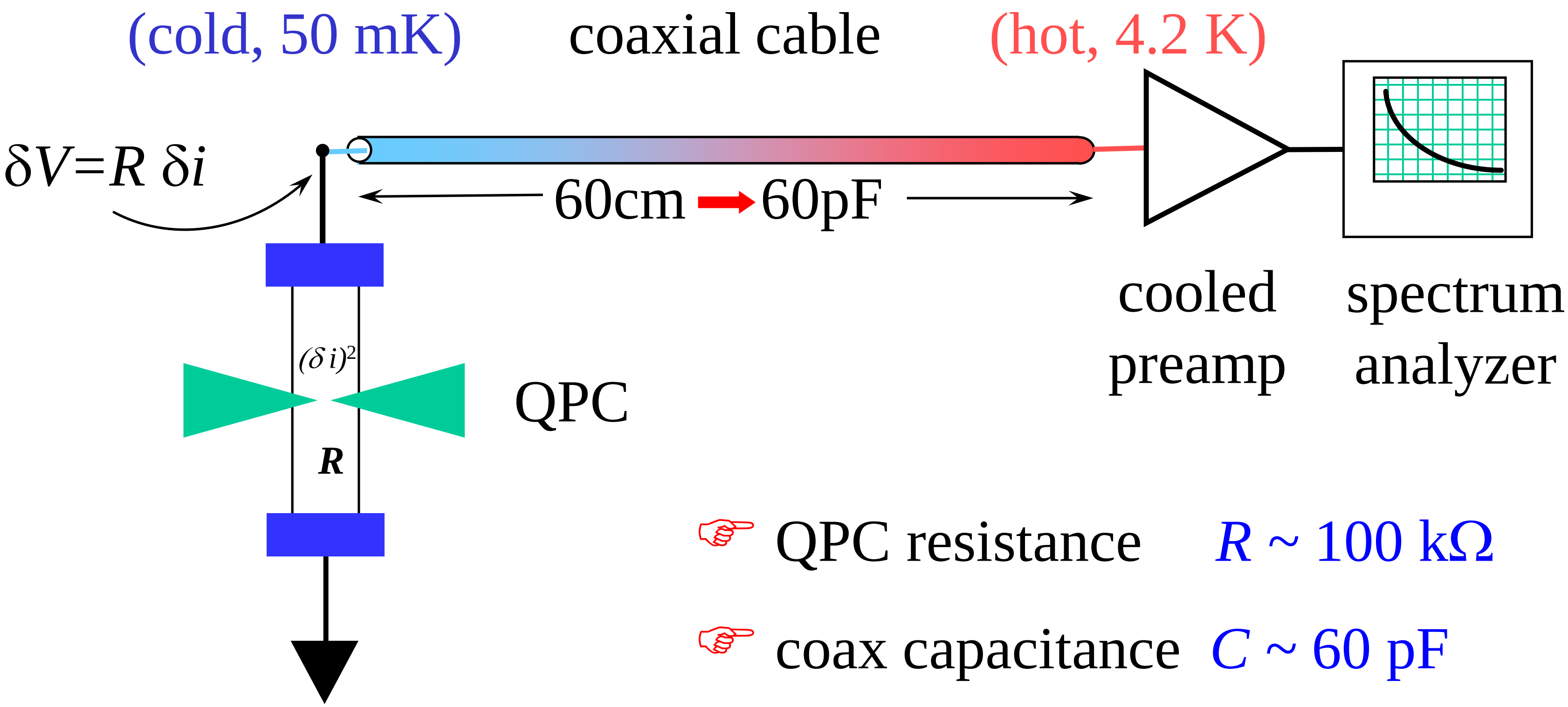
Johnson noise.. T ~ (10-30) mK

noise in 'warm electronics'...T^*~ 3.5 K

"home made" (MODFET) cryogenic preamplifier (T= 4.2 K)

T^*~100-200 mK at f_0 = 1 - 4 MHz (above $1/f$ noise knee)

difficulties in measurements



☞ QPC resistance $R \sim 100\ \mathrm{k\Omega}$

☞ coax capacitance $C \sim 60\ \mathrm{pF}$

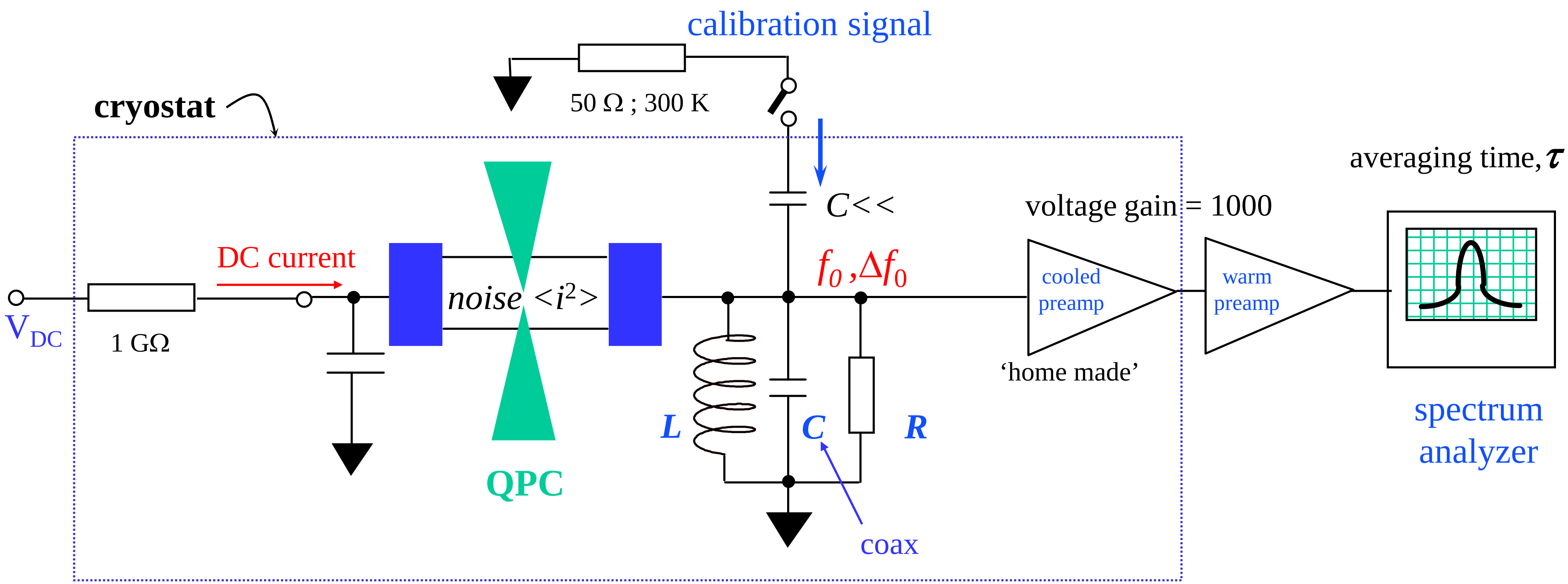
⬇

$$f_{max} = 1/(2\pi RC) \sim 30\ \mathrm{kHz}$$

∴ $1/f$ noise is large

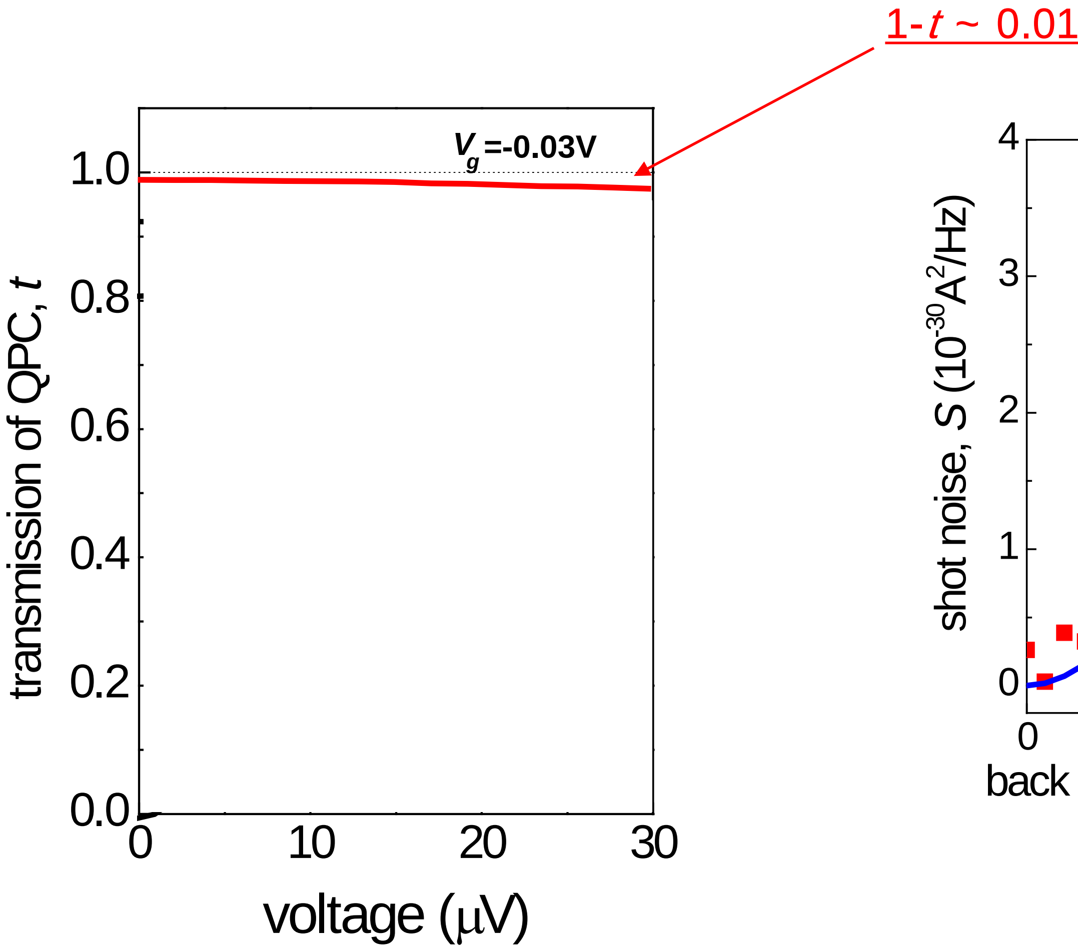
experimental setup

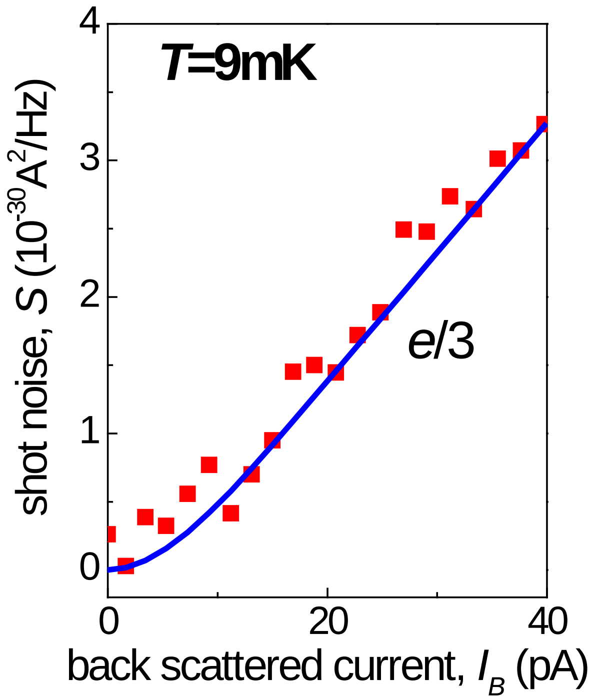
* frequency above *1/f* noise corner of preamplifier;
* capacitance compensated by resonant circuit;



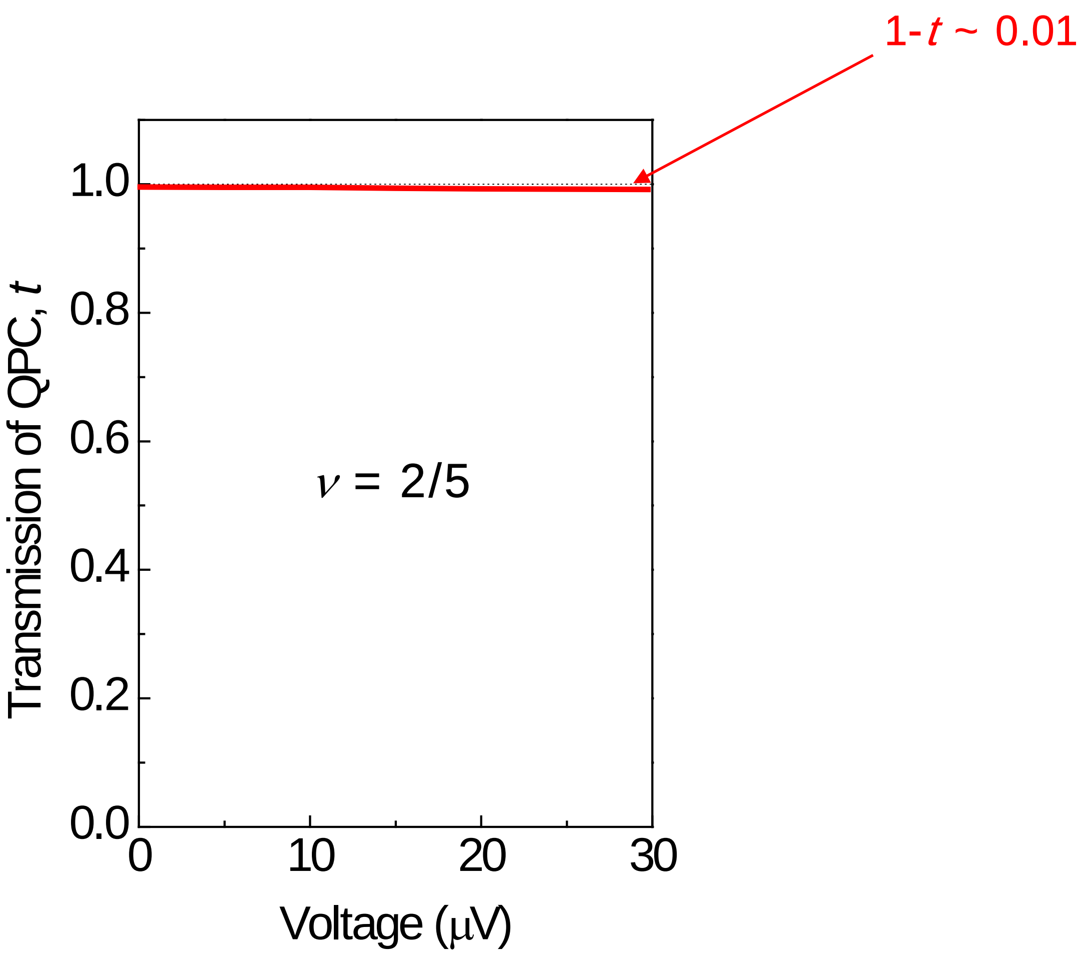
$$f_0 = \frac{1}{2\pi\sqrt{LC}} \approx 2 \rightarrow 4MHz \quad , \quad \Delta f_0 = \frac{1}{2\pi RC} \approx 30\,kHz$$

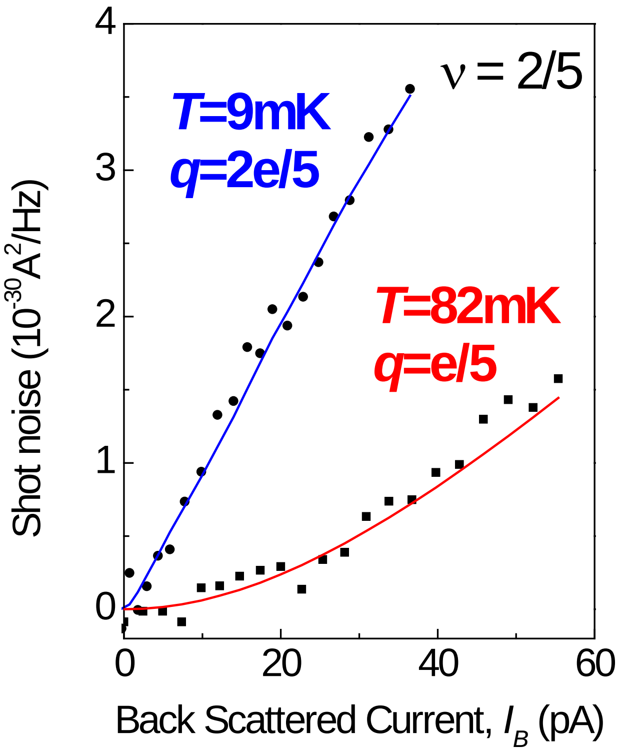
$\nu = 1/3$


1-*t* ~ 0.01
V_g=-0.03V
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
voltage (μV)
transmission of QPC, *t*


T=9mK
e/3
4
3
2
1
0
0
20
40
back scattered current, I_B (pA)
shot noise, *S* (10^{-30}A^2/Hz)

$\nu = 2/5$ *bunching*

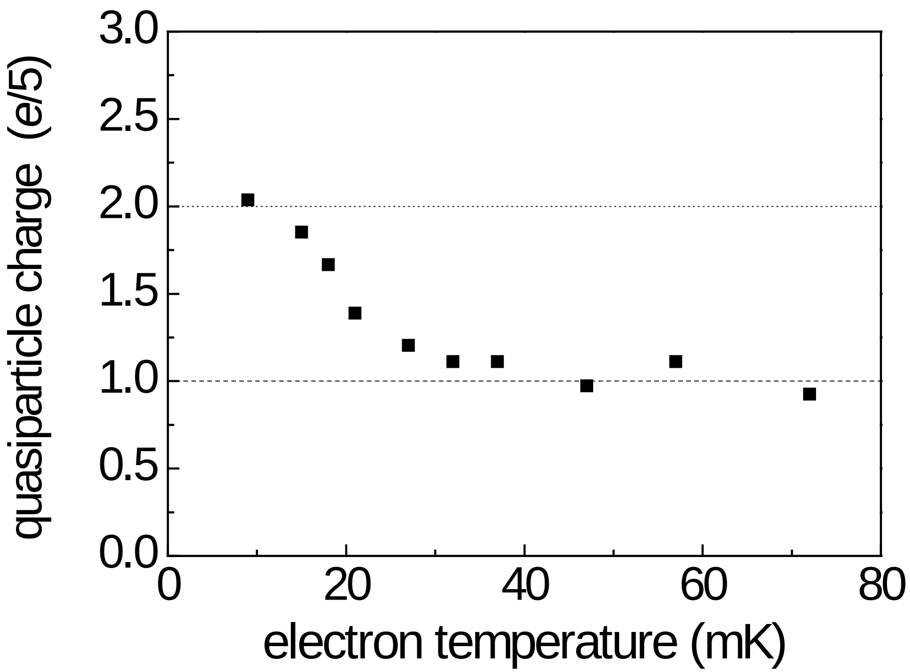




bunching of quasiparticles

charge = filling factor

temperature dependence $\nu = 2/5$

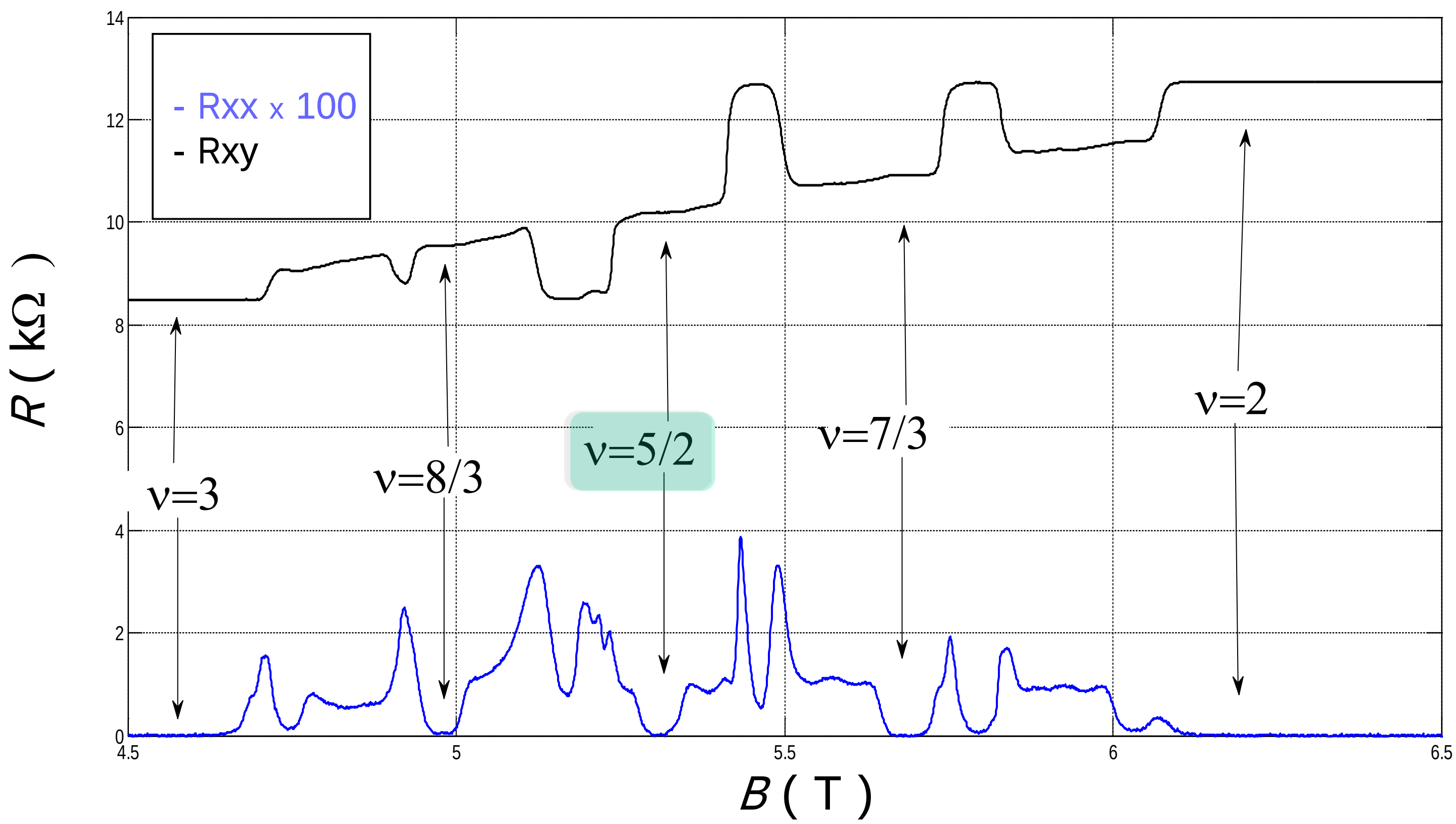


bunching of quasiparticles at low temperatures

similar effect at ν=3/7...

1^{st} excited Landau level

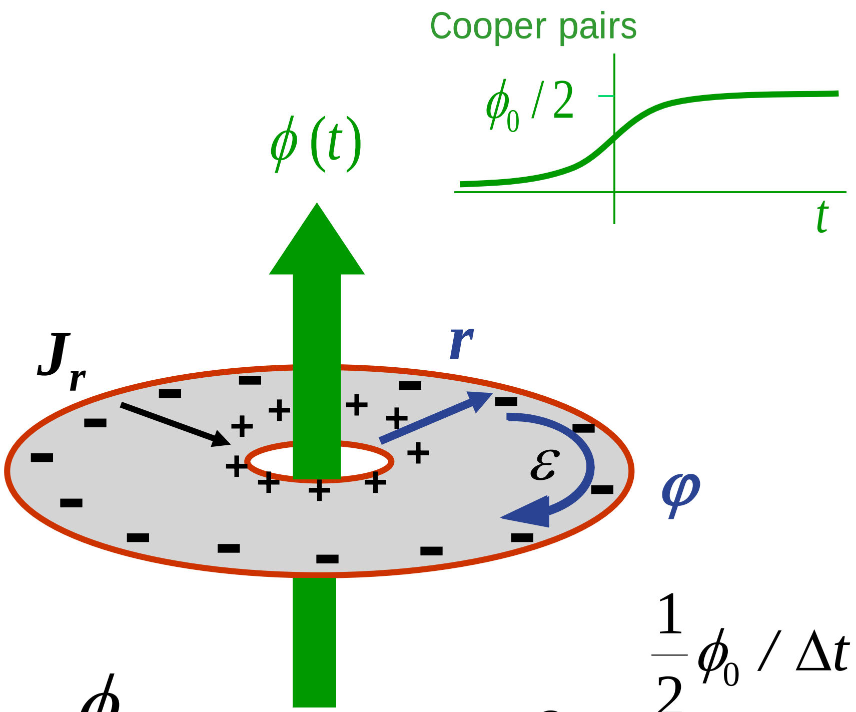
ν =5/2   Moore - Read proposed non-abelian state



μ ~ 30·10^6 cm^2/V-s

expected charge of excitations

$$\sigma_{xx} = 0 \qquad \sigma_{xy} = \frac{1}{2}\frac{e^2}{h}$$



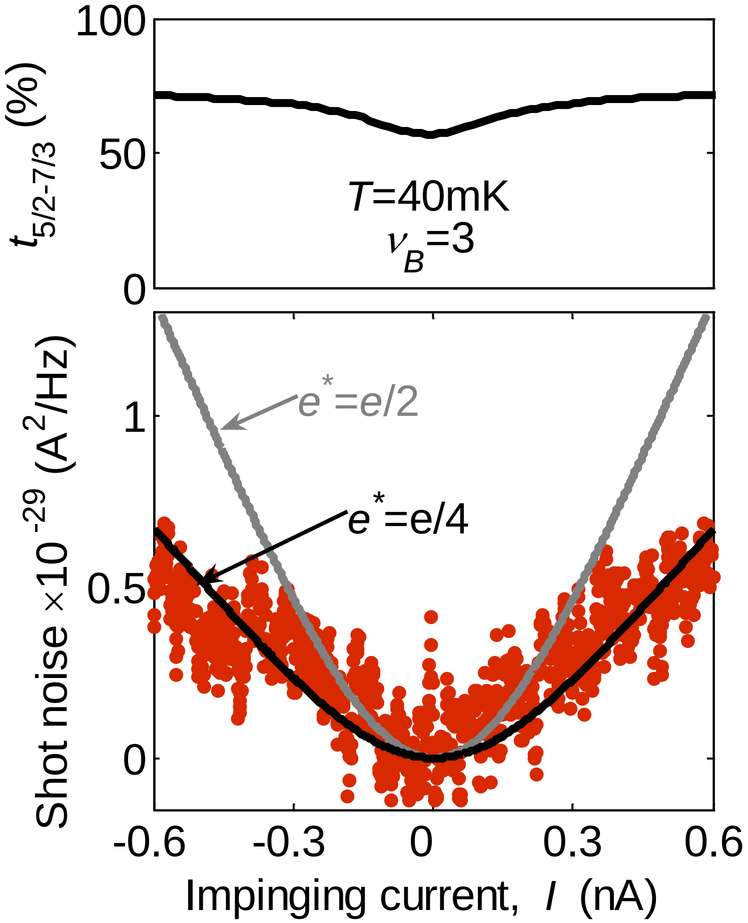
$$\varepsilon_\varphi = \frac{\frac{1}{2}\phi_0 / \Delta t}{2\pi r}$$

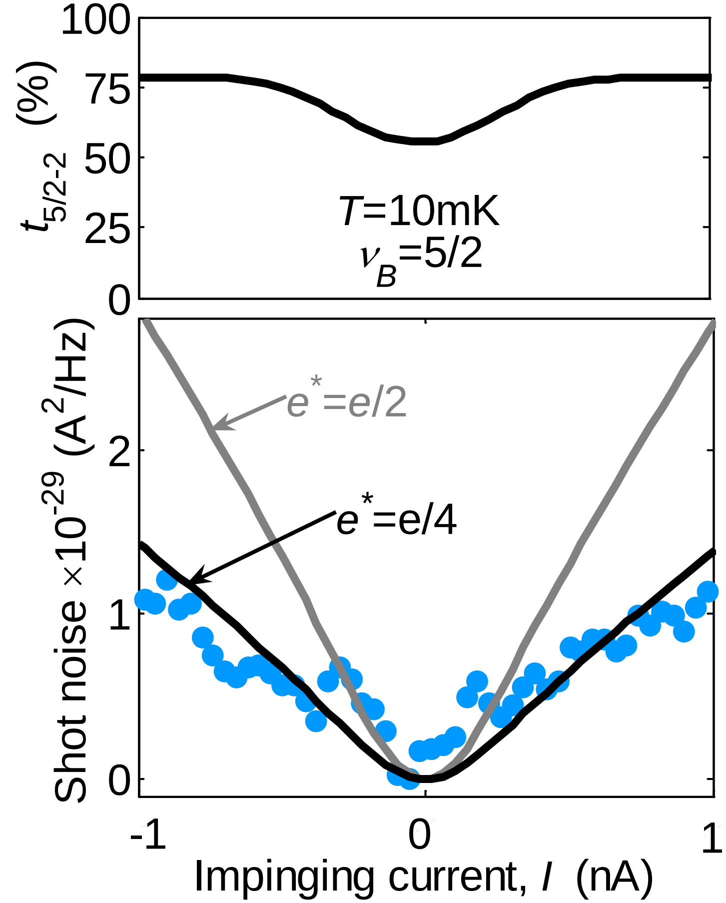
$$q(t) = \int_{\Delta t} dt \cdot J_r \cdot 2\pi r = \sigma_{xy}\frac{\phi_0}{2}$$

$$J_r = \sigma_{xy}\varepsilon_\varphi$$

$$e^* = e/4$$

this fractions is fragile...

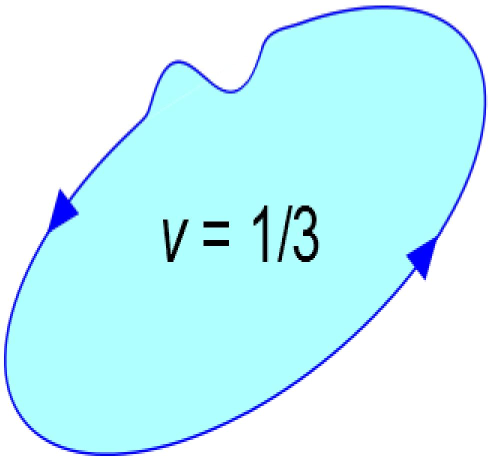


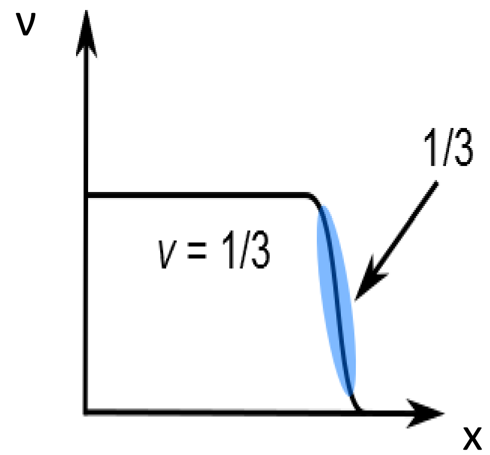


particle-like (Laughlin) *vs* hole-conjugate

quasiparticles

expected ν = 1/3 particle-like





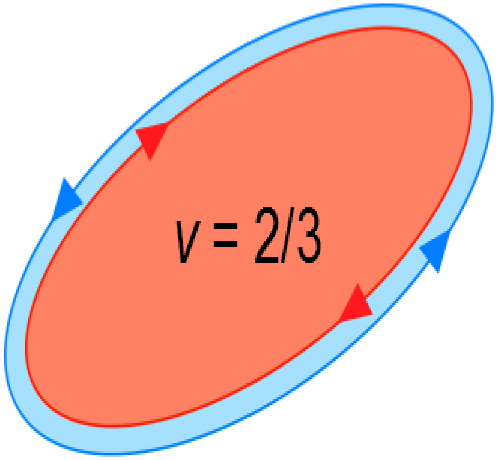
bulk: single component and gapped (incompressible liquid)

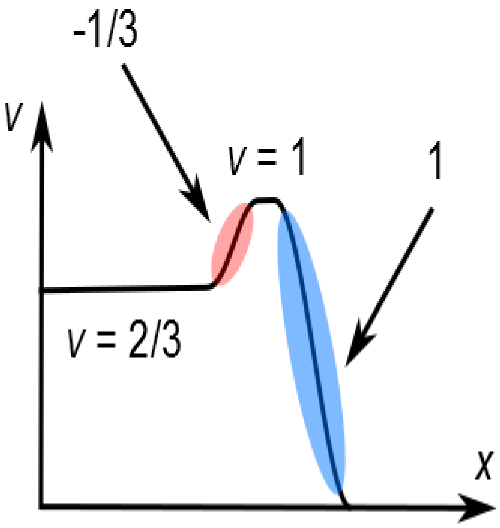
edge: single charge mode with $G=G_0/3$

$\nu = 2/3$ hole-conjugate

$\nu = 2/3 = 1 - 1/3$

full LL of electrons – 1/3 holes

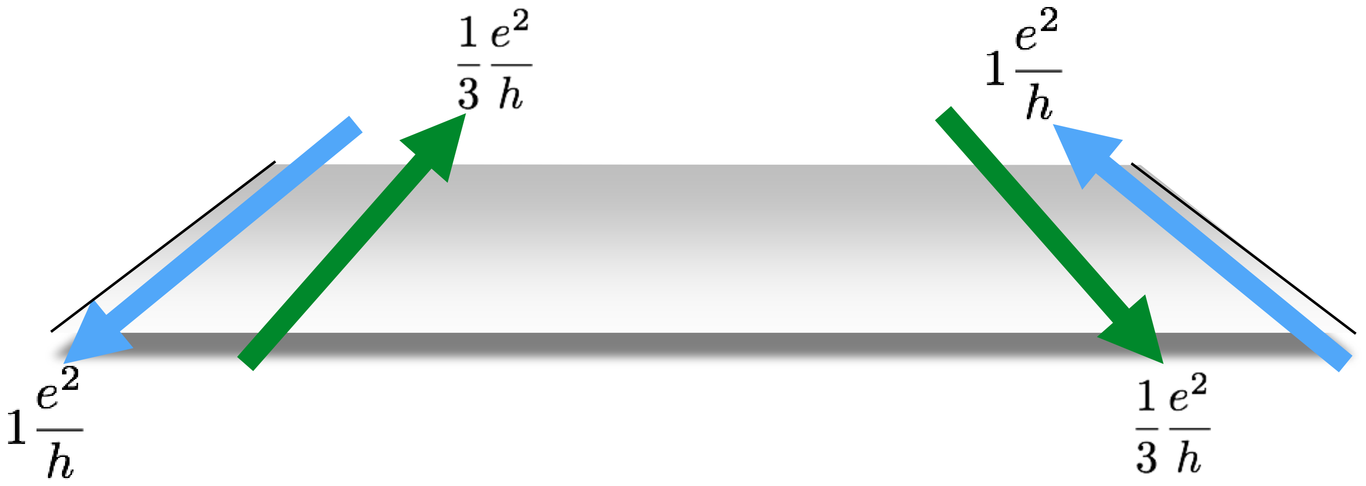




Macdonald (1992)

upstream e/3 was not found…..Ashoori 1992

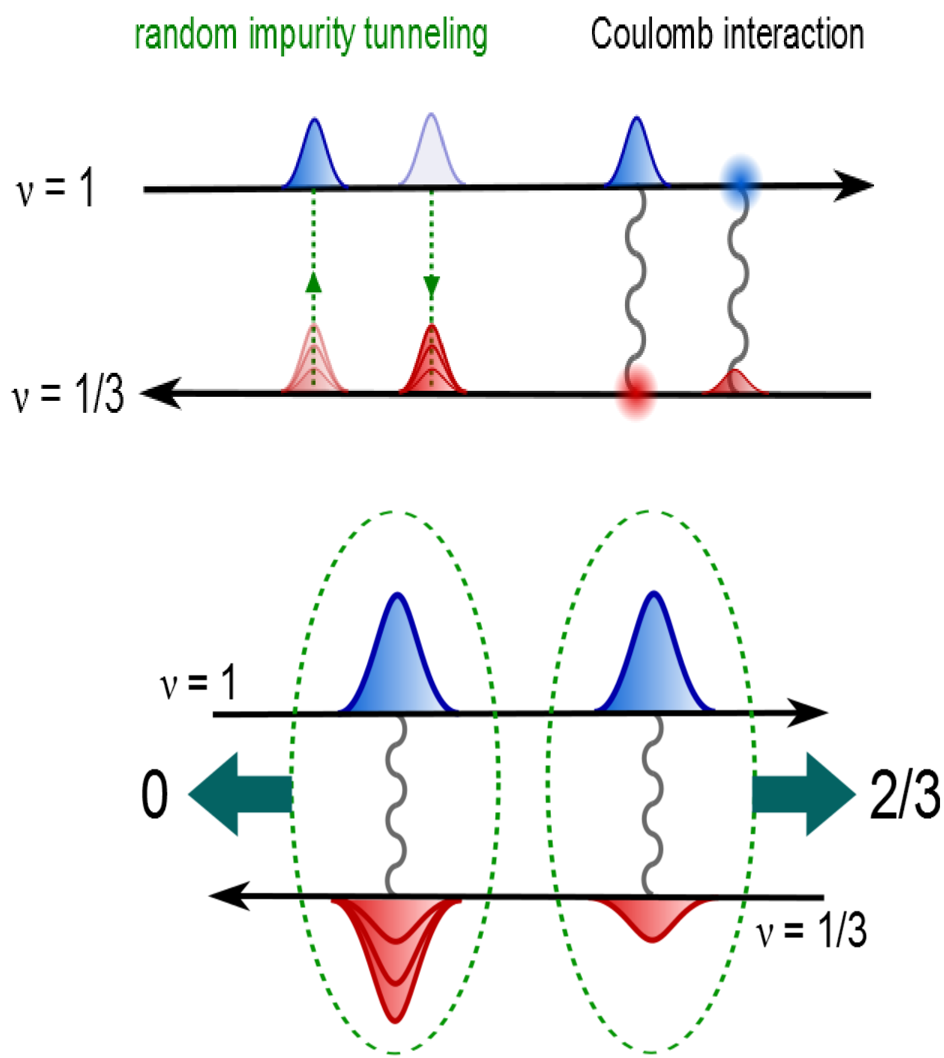
MacDonald's clean edge



naive model 2-probe conductance = 4/3 e^2/h

measured 2-probe conductance = 2/3 e^2/h

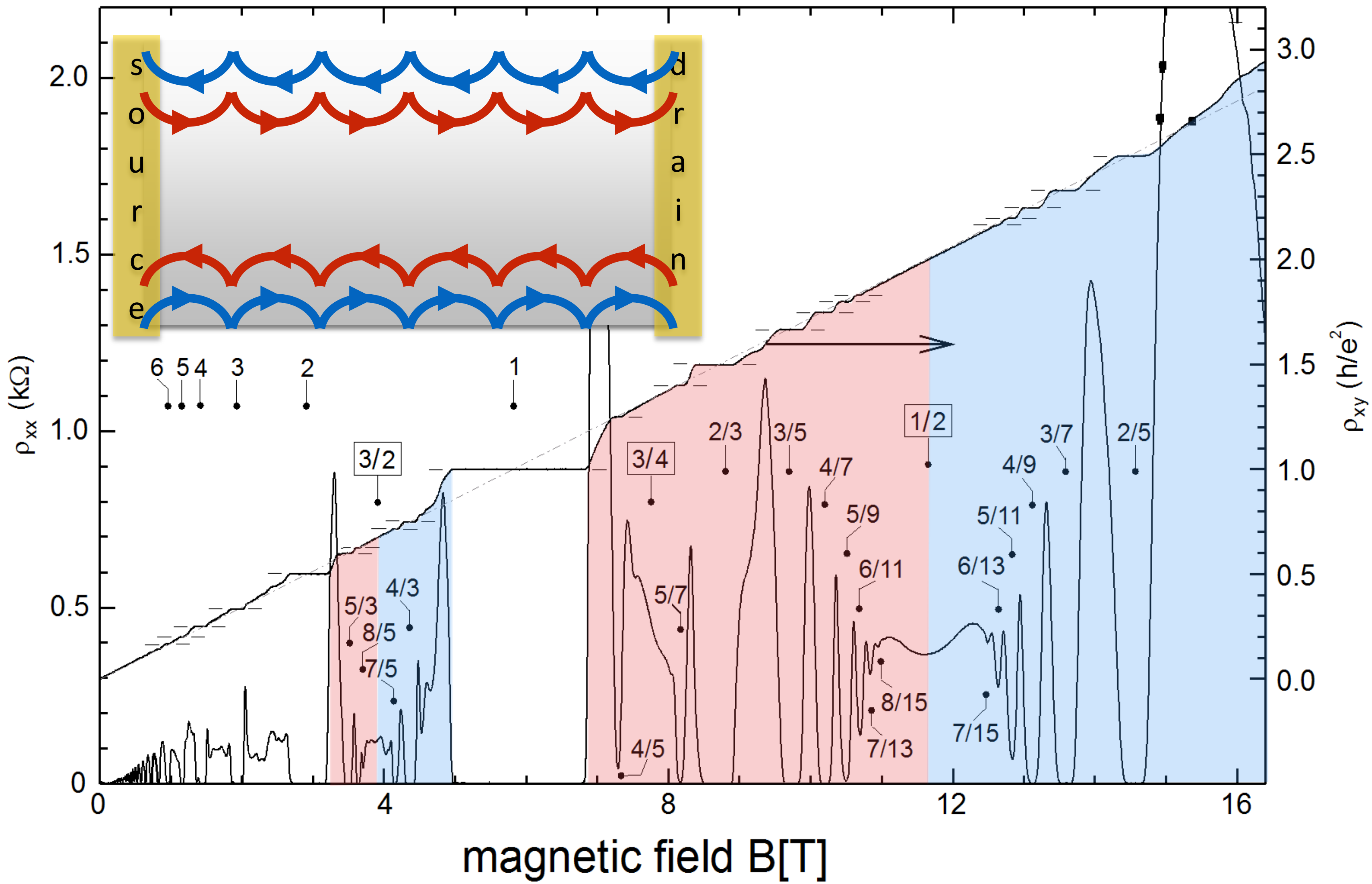
upstream *neutral* mode @ ν =2/3



downstream charge mode 2/3 e^2/h + *upstream* neutral mode

Kane et al. 1994

hole-conjugate states



edge modes mirror the bulk

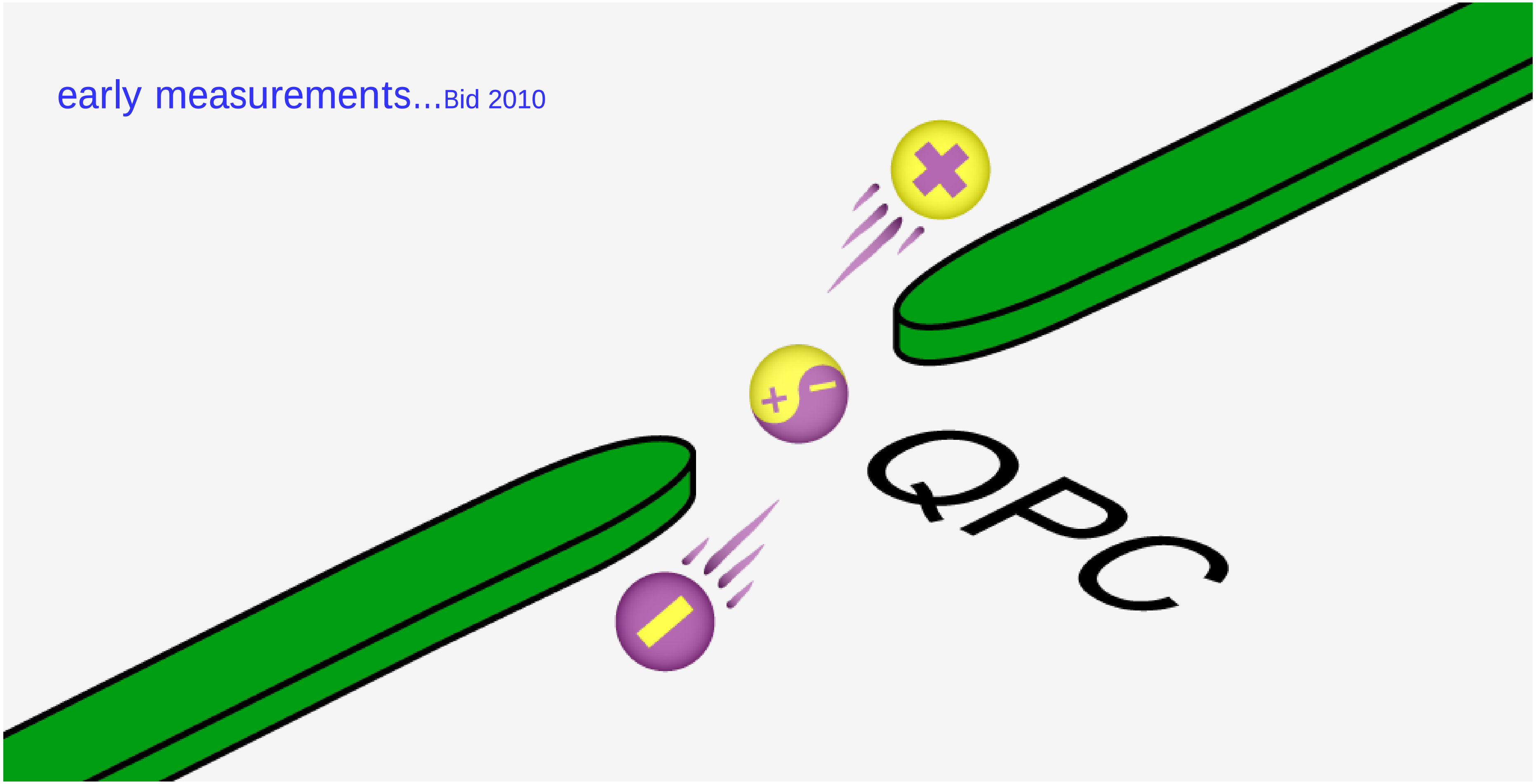
‘*bulk – edge*’ correspondence

net number of modes (down minus up) – must be preserved in equilibration

neutral modes

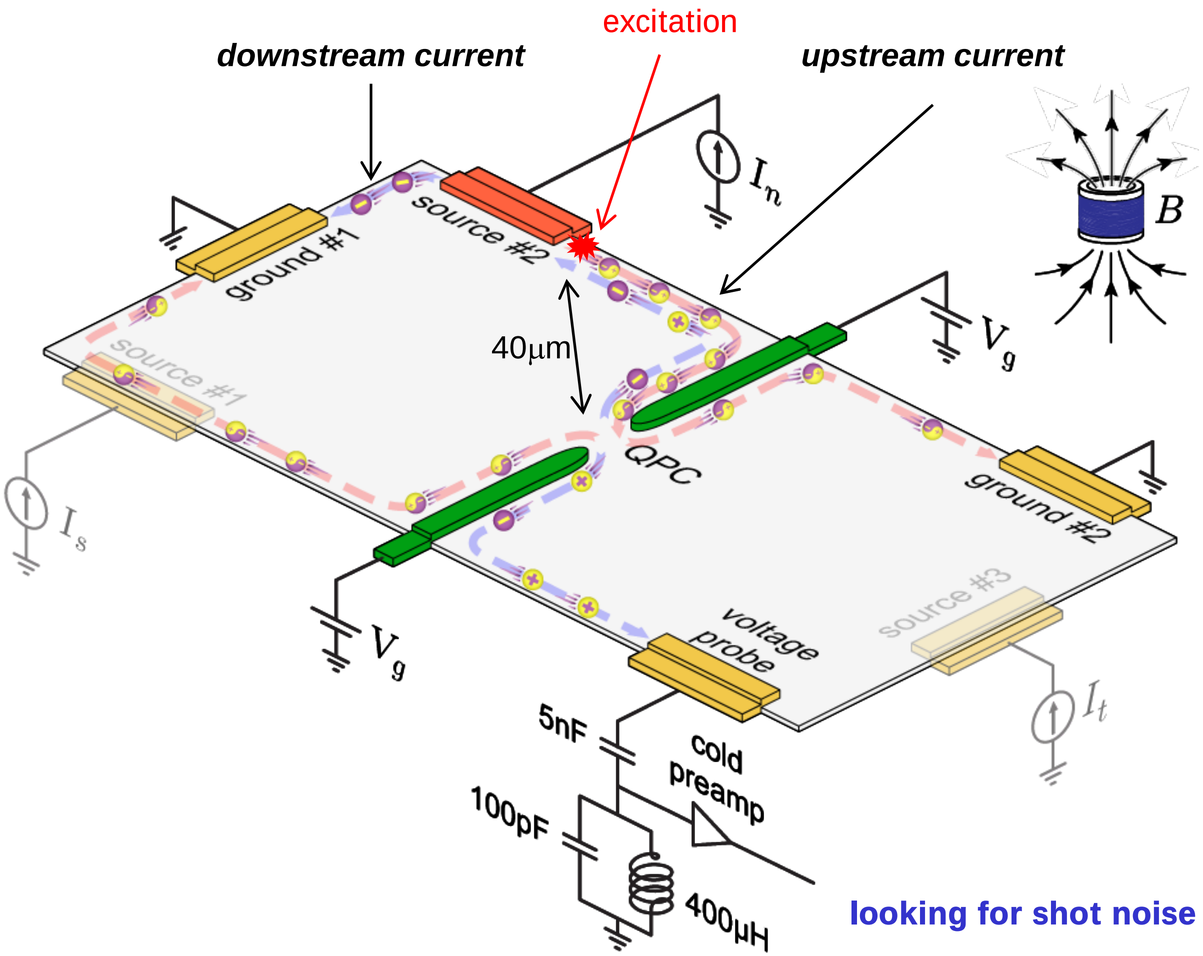
- not observed in charge transport measurements
- carrying energy without net charge dipole like
- possible source of dephasing of interference
- topological .... or due to edge-reconstruction

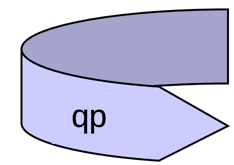
neutral mode → ‘flow of dipoles’



shot noise (electron-hole) without net current

excitation of neutral mode hot spot



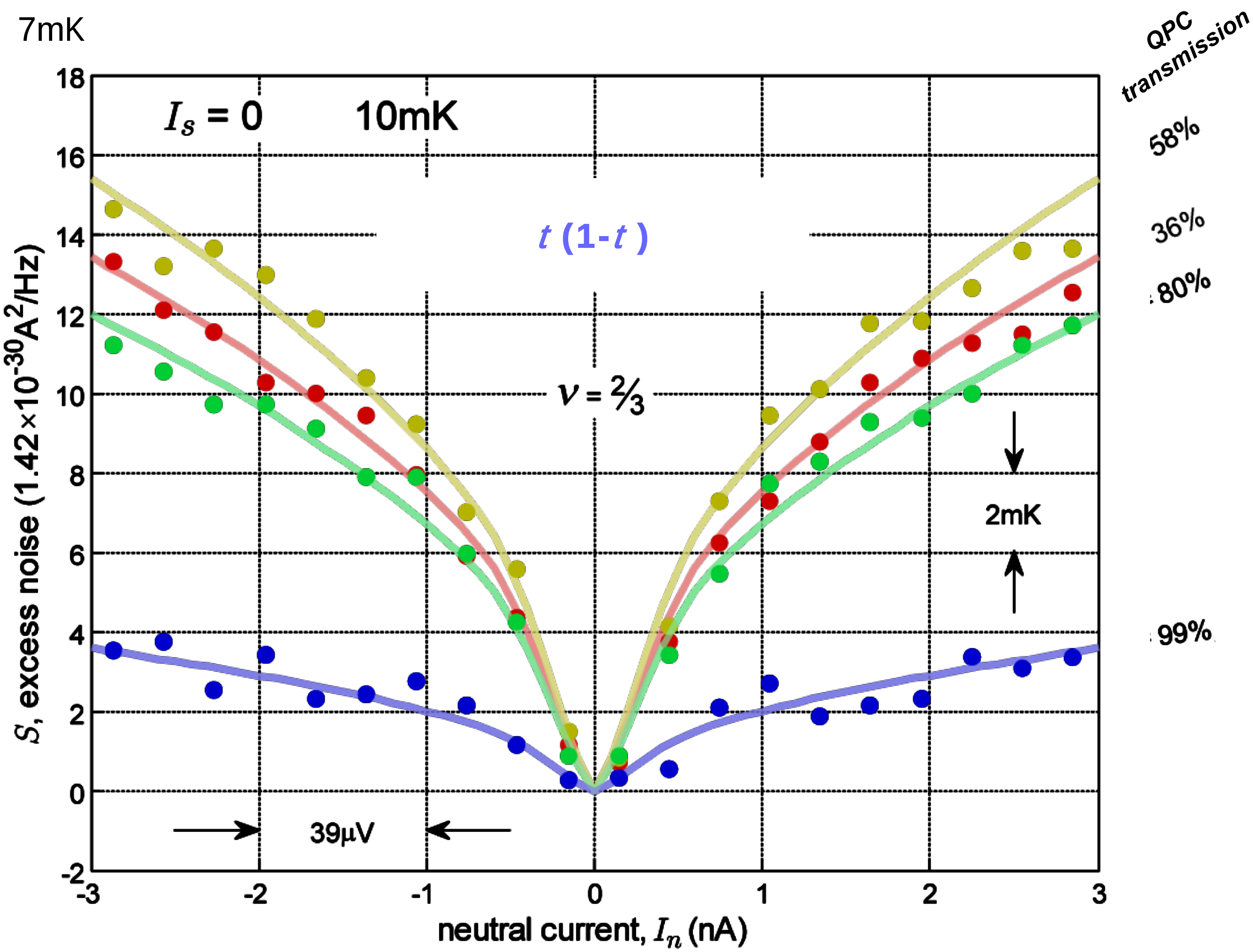


looking for shot noise

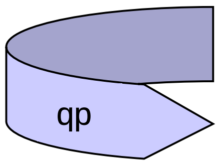
with zero net current

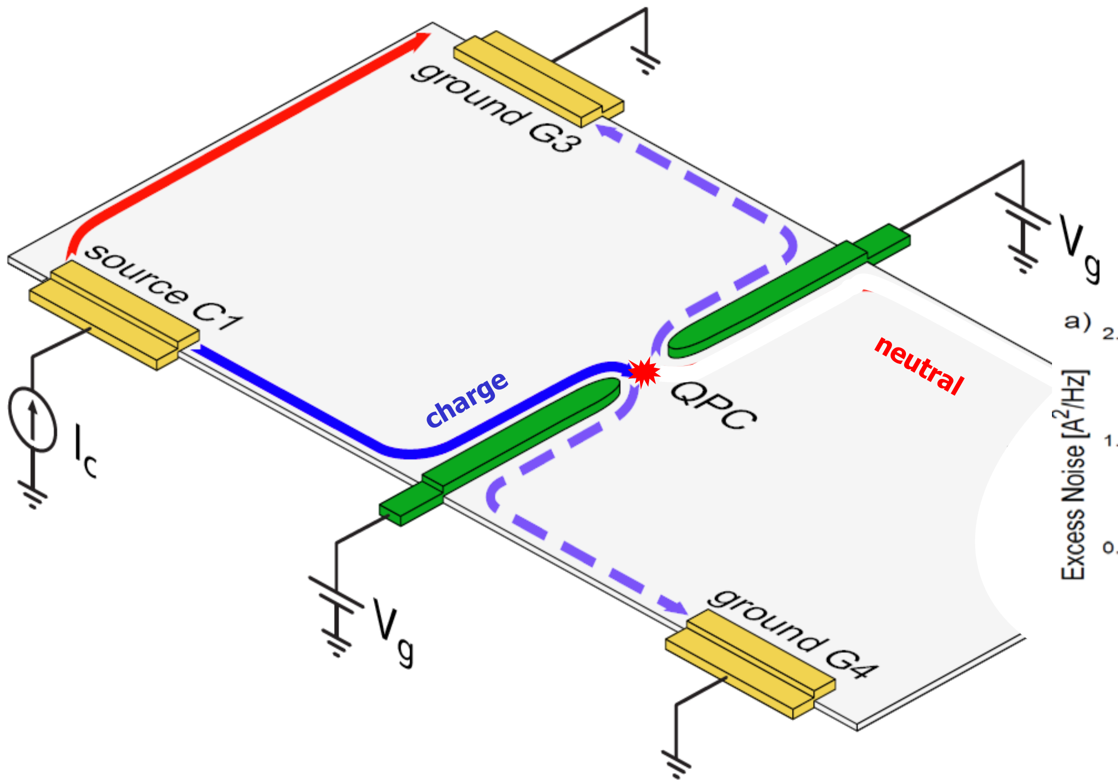
upstream noise $\nu = 2/3$

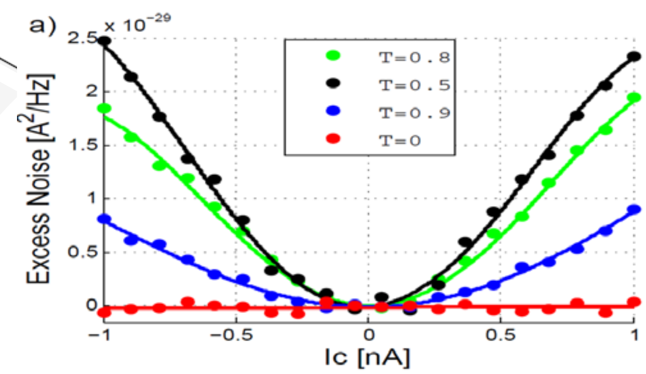
$10^{-29}A^2/Hz \rightarrow$ 7mK

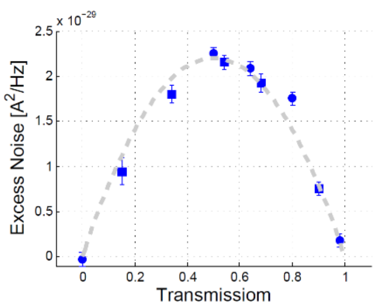


excitation of neutral mode at QPC …



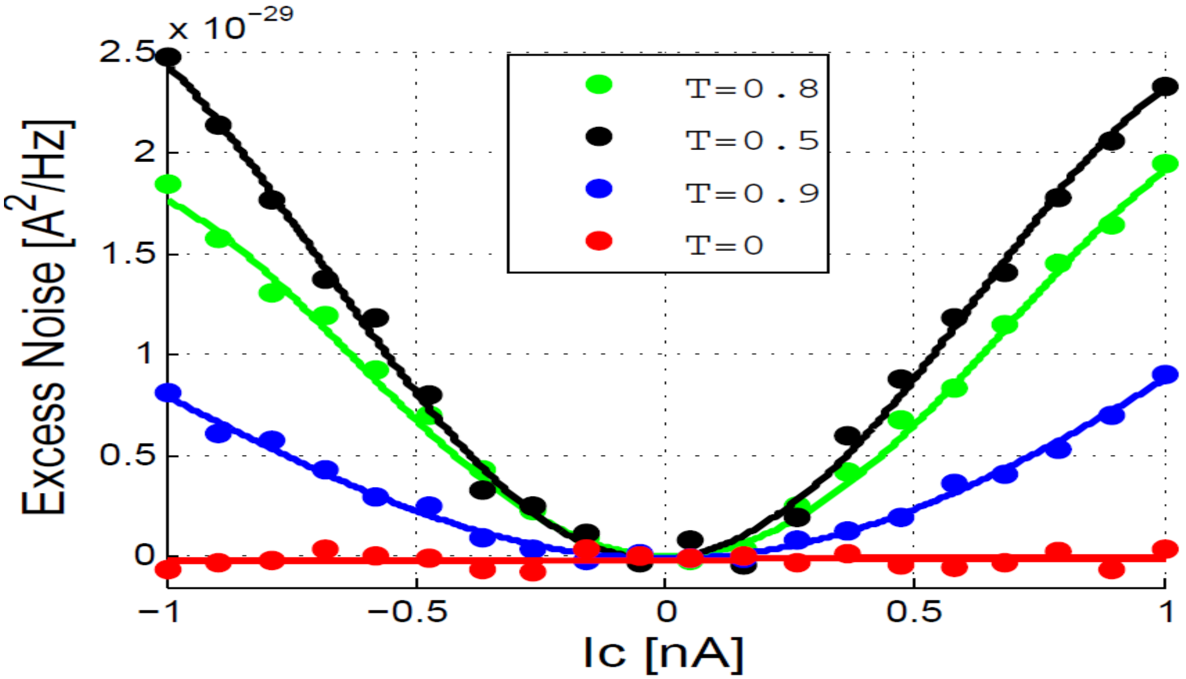


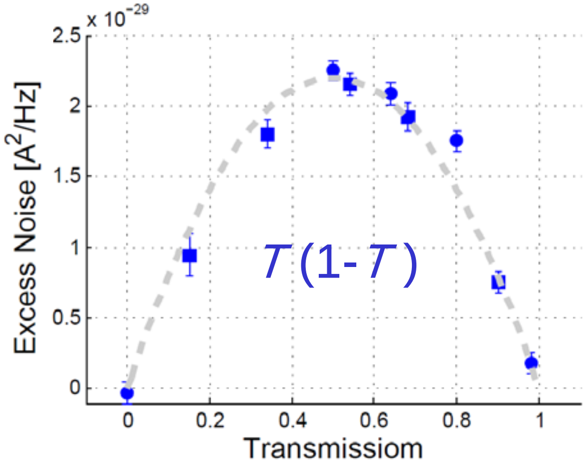




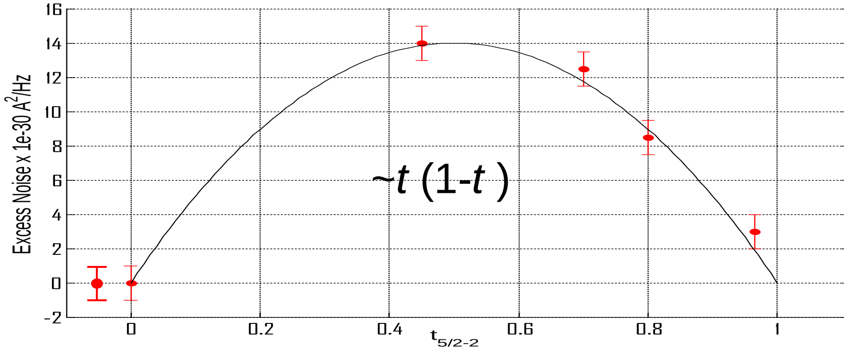
upstream noise $\nu = 2/3$

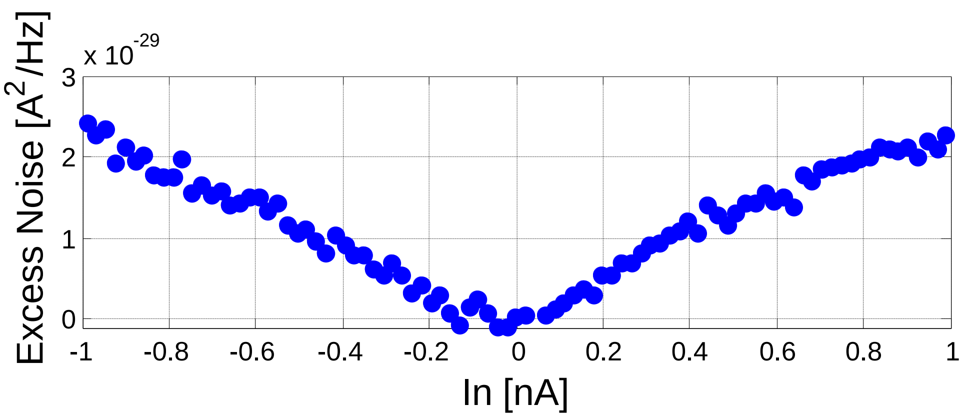
noise due to neutral mode

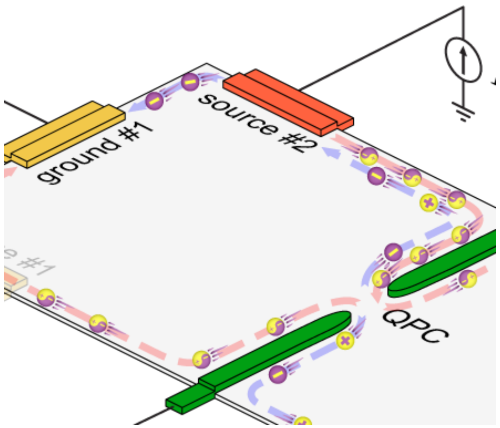




neutral mode at $\nu = 5/2$







clear evidence of upstream neutral mode

ν = 5/2 ….. two leading orders

state	statistics	charge	upstream neutral mode
Moore-Read (Pfaffian) Moore & Read, Nuclear Phys. B (1991)	non-abelian	e/4	no
anti-Pfaffian Lee, PRL (2007); Levin et. al. PRL (2007)	non-abelian	e/4	yes

there are other orders too…..will see later

to be continued